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КОСМОЛОГИЯ НЕОДНОРОДНОЙ ВРАЩАЮЩЕЙСЯ ВСЕЛЕННОЙ И ПРЕДСТАВЛЕНИЕ РЕАЛЬНОСТИ

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На основе решений уравнений Эйнштейна предложены универсальные метрики, описывающие свойства неоднородной вращающейся Вселенной. Показано, что в случае метрики пространства-времени отрицательной кривизны, уравнения поля имеют гиперболический, параболический или эллиптический тип, в зависимости от уравнения состояния. Предложено уравнение состояние общего вида, описывающее возникновение материи, как агрегатного состояния темной энергии

Ключевые слова: : МЕТАГАЛАКТИКА, ГАЛАКТИЧЕСКИЙ КЛАСТЕР, ГЕОМЕТРИЯ ПРОСТРАНСТВА-ВРЕМЕНИ, ТЕОРИЯ ГРАВИТАЦИИ ЭЙНШТЕЙНА, КОСМОЛОГИЯ, ТЕМНАЯ МАТЕРИЯ, ТЕМНАЯ ЭНЕРГИЯ.

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COSMOLOGY OF INHOMOGENEOUS ROTATING UNIVERSE AND REALITY SHOW

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On the basis of the solutions of Einstein we have proposed universal metric describing the properties of an inhomogeneous rotating universe. It is shown that in the case of space-time metric of negative curvature, the field equations are hyperbolic, parabolic or elliptic type, depending on the equation of state. An equation for the general state of the species, describing the emergence of matter, as the aggregate state of dark energy proposed

Keywords: COSMOLOGY, GENERAL RELATIVITY, GALACTIC CLUSTER, SPACE-TIME GEOMETRY, BLACK MATTER, BLACK ENERGY.

Introduction

General Relativity [1-3] is widely used in modern cosmology, especially in connection with the discovery of the accelerated expansion of the Universe [4].

Einstein's gravitational field equations have the form [1]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = g_{\mu\nu}\Lambda + \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

$R_{\mu\nu}, g_{\mu\nu}, T_{\mu\nu}$ - The Ricci tensor, the metric tensor and energy-momentum tensor; Λ, G, c - Einstein's cosmological constant, the gravitational constant and the speed of light, respectively.

Note that the energy-momentum tensor cannot be used directly in the construction of the geometry due to the uncertainty of the concept of matter , which should be defined, for example, in the framework of geometrodynamics [5-8].

In general case, we have the relations

$$R_{ik} = R^{j}_{ijk}, \quad R = g^{ik} R_{ik},$$
$$R^{\alpha}_{\beta\gamma\delta} = \frac{\partial \Gamma^{\alpha}_{\beta\delta}}{\partial x^{\gamma}} - \frac{\partial \Gamma^{\alpha}_{\beta\gamma}}{\partial x^{\delta}} + \Gamma^{\mu}_{\beta\delta}\Gamma^{\alpha}_{\mu\gamma} - \Gamma^{\mu}_{\beta\gamma}\Gamma^{\alpha}_{\mu\delta}, \quad (2)$$
$$\Gamma^{i}_{jk} = \frac{1}{2} g^{is}\left(\frac{\partial g_{sj}}{\partial x^{k}} + \frac{\partial g_{sk}}{\partial x^{j}} - \frac{\partial g_{jk}}{\partial x^{s}}\right)$$

$R^{\alpha}_{\beta\gamma\delta}$ - Riemann tensor, Γ^{i}_{kl} - Christoffel symbols of the second kind.

As is well known, Einstein proposed in 1912-1955 several alternative theories of gravity, including the theory of (1) has been universally recognized. Cosmological constant introduced by Einstein in 1917 in [1] caused much controversy. However, the origin of this effect is related to one of the great mysteries of modern physics [9-13].

Indeed, this term could arise as a consequence of quantum fluctuations, but the corresponding estimates show that there is a huge difference, are 120 orders of magnitude between the experimental and predictive theory of quantum gravity. This difference can be somewhat reduced by using various considerations [9], but cannot be eliminated.

Marked a huge difference between facts and theory indicates that the geometry between the microcosm and the geometry across the universe there is no connection. But then Einstein's equation (1) loses its meaning, since matter is composed of atoms and elementary particles. To resolve this contradiction was formulated principle of maximum certainty: space-time metric depends only on such fundamental constants, which are determined as accurately as possible.

Note that in modern physics such constants are the speed of light, Planck's constant, the fine structure constant, the mass of the electron, the proton mass, and some other value. Energy-momentum tensor of matter has a relatively low accuracy of the determination, and with the high content of dark matter in the observable universe, all is not defined, so it cannot enter the number of parameters that affect the space-time metric.

To keep the basic idea of the metric in the Einstein theory of gravity and thus satisfy the principle of maximum certainty, we assume that the Einstein equation (1) splits into two independent equations [14-16]:

$$\begin{aligned} & R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \lambda g_{\mu\nu} = 0 \\ & g_{\mu\nu}(\Lambda + \lambda) + \frac{8\pi G}{c^4} T_{\mu\nu} = 0 \end{aligned} \qquad (3)$$

Here λ - a function that depends on the measurement of fundamental constants available as accurately as possible. Note that the first equation is determined by the space-time, and the second equation is given by the distribution of matter, which corresponds to this metric.

With this approach, there is no need to build hypotheses regarding the distribution of mass and energy in the universe. The basis of all of the observed phenomena is metric, which corresponds to the distribution of mass and energy, determined from the second equation (2). Metric depends only on the fundamental constants, and the distribution of mass and energy is completely determined by the geometry, which is consistent with the mechanism of mass generation in the standard model, and in the quantum theory of gravity. Obviously, it is not necessary to deduce the mechanism of some other physical phenomena, as all these phenomena are already reflected in the metric of space-time , which is not only the scene of all the events, but also their cause and effect [5-8]. Matter in the model (2) is a passive component, the presence of which is not mandatory. This can be compared with the passage of colored water, in which the ink is a passive component that allows visualization of the movement, but does not influence the motion [14-17].

In the works [15-16] was built metric local Supercluster of galaxies based on axially symmetric solutions of the model (3). In the paper [17] we considered a model of an inhomogeneous rotating universe. To find solutions to the first equation (3), consistent with the standard cosmological model without hypotheses on the distribution of matter and energy, were selected metrics with the exact value of parameter $\lambda = -1$.

In this paper, we construct a metric that can be used to represent reality, containing areas in which the field equations are hyperbolic, parabolic or elliptic type. It is shown that the field equations of mixed type have solutions describing gravitational geon in the space of negative curvature with the exact value $\lambda = -1$. On the basis of these decisions, a model generation of matter from dark energy by the phase transition is proposed.

Metric inhomogeneous rotating universe

For modeling the rotation of the universe we use the known results [18-21] and metric [22] modified based on the separation of angular variables and distance to objects in the form of

$$\begin{aligned} &ds^2 = dt^2 + 2b(t,r)dtdr - a^2(t,r)dr^2 - d\vartheta^2 - \sin^2\vartheta d\phi^2 \\ &g_{00} = 1,\ g_{11} = -a^2(t,r),\ g_{01} = g_{10} = b(t,r) \\ &g_{02} = g_{13} = g_{23} = g_{03} = 0, \\ &g_{22} = -1,\ g_{33} = -\sin^2\vartheta \end{aligned} \tag{4}$$

Here $a = a(t,r), b = b(t,r)$ - functions satisfying the Einstein equations. Such a choice of metric is due primarily to the fact that in modern astrophysics there is no reliable method of determining distances on the scale of the observable universe. Spherical coordinate system used for the compilation of astronomical catalogs, defined on the unit sphere, and the distance to the object is some functional, depending on the use of physical models [23-24], and not on the coordinates.

Nonzero components of the Einstein tensor in the metric (4) have the form

$$\begin{aligned} &G_{00} = 1 \\ &G_{01} = G_{10} = b(t,r) \\ &G_{11} = -a^2(t,r) \\ &G_{22}\sin^2\vartheta = G_{33} \\ &G_{22} = \frac{bb_t(b_r + aa_t) - b^2(a_t^2 + b_{tr} + aa_{tt}) + aa_r b_t - a^2(b_{tr} + aa_{tt})}{(a^2+b^2)^2} \end{aligned} \quad (5)$$

Assuming that $G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = -\lambda g_{\alpha\beta}$, we find the field equations. From the first three conditional expressions (5) and the form of the metric tensor implies that in the metric (4) the parameter $\lambda = -1$. Consequently, the scalar curvature of the test of Einstein spaces is a negative $R = -4$.

Of the two functions, $a = a(t,r),\ b = b(t,r)$ only one function can be determined from the equations of the field, while the other should be set. Suppose, $b = const$, then the metric (4) can be generalized to cases corresponding to the field equations of hyperbolic, elliptic or mixed type, respectively.

The field equations of hyperbolic type

Consider the metric of the form

$$\begin{aligned} &ds^2 = \psi(t,r)dt^2 + 2bdtdr - \psi(t,r)dr^2 - d\vartheta^2 - \sin^2\vartheta d\phi^2 \\ &g_{00} = \psi(t,r),\ g_{11} = -\psi(t,r),\ g_{01} = g_{10} = b \\ &g_{02} = g_{13} = g_{23} = g_{03} = 0, \\ &g_{22} = -1,\ g_{33} = -\sin^2\vartheta \end{aligned} \quad (6)$$

Nonzero components of the Einstein tensor in the metric (6) have the form

$$\begin{aligned} &G_{00} = \psi(t,r),\ G_{01} = G_{10} = b, \\ &G_{11} = -\psi(t,r),\ G_{22}\sin^2\vartheta = G_{33}, \\ &G_{22} = \frac{\psi(\psi_t{}^2 - \psi_r{}^2) - (b^2+\psi^2)(\psi_{tt} - \psi_{rr})}{2(\psi^2+b^2)^2} \end{aligned} \quad (7)$$

Assuming $G_{22} = -1$ we find the hyperbolic equation to determine the metric

$$\frac{\psi(\psi_t^2 - \psi_r^2) - (b^2 + \psi^2)(\psi_{tt} - \psi_{rr})}{2(\psi^2 + b^2)^2} = -1 \qquad (8)$$

The model (8) can solve the problem of the decay of clusters of galaxies. We define the metric in the initial state as a normal distribution along the radial coordinate $\psi(0,r) = \psi_0 + \exp(-r^2)$. Then by equation (8) over time cluster splits into separate clusters, forming a halo, which correspond to the resulting nonlinear wave crests - Fig. 1. This decay depends on the parameters b and ψ_0.

Note that this type of scalar waves may be responsible for the formation of individual galaxies, and even elementary particles, such as scalar bosons. Indeed, the equation (8) does not contain any scale parameters, except for the time and location, the extent of which can be chosen arbitrarily.

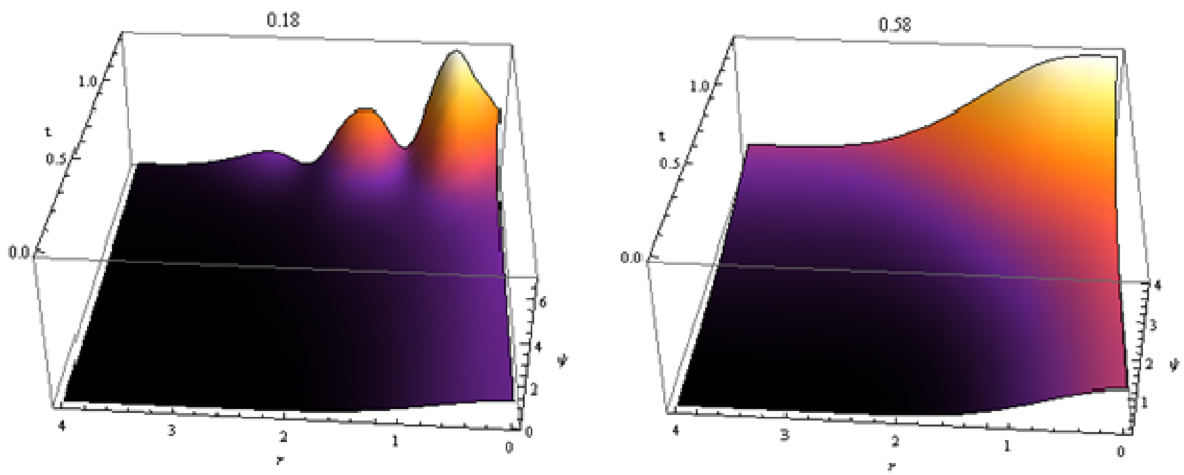

Figure 1: Cluster decay into a number of clusters in the model (8). Initial perturbation is given in the form $\psi(0,r) = 0.1 + \exp(-r^2)$. Above picture shows the value of the parameter b.

The field equations of elliptic type

Consider the metric of the form

$$\begin{aligned} & ds^2 = \psi(t,r)dt^2 + 2bdtdr - dr^2/\psi(t,r) - d\vartheta^2 - \sin^2\vartheta d\phi^2 \\ & g_{00} = \psi(t,r),\ g_{11} = -1/\psi(t,r),\ g_{01} = g_{10} = b \\ & g_{02} = g_{13} = g_{23} = g_{03} = 0, \\ & g_{22} = -1,\ g_{33} = -\sin^2\vartheta \end{aligned} \tag{9}$$

Calculating the nonzero components of the Einstein tensor in the metric (9), we find

$$\begin{aligned} & G_{00} = \psi(t,r),\ G_{01} = G_{10} = b, \\ & G_{11} = -1/\psi(t,r),\ G_{22}\sin^2\vartheta = G_{33}, \\ & G_{22} = \frac{-2\psi_t^2 + \psi\psi_{tt} + \psi^3\psi_{rr}}{2(1+b^2)\psi^3} \end{aligned} \tag{10}$$

Using the equation $G_{22} = -1$, we have

$$\psi\psi_{tt} + \psi^3\psi_{rr} = -2(1+b^2)\psi^3 + 2\psi_t^2 \tag{11}$$

In a case of static metrics set $\psi_t = \psi_{tt} = 0$, then integrating the equation (11), we find

$$\psi = k_1 + k_2 r - (1+b^2)r^2 \tag{12}$$

Here k_1, k_2 are arbitrary constants. As is known, the quadratic potential (12) leads to a divergence of galaxies in accordance to Hubble law [15-16].

Note that equation (12) is an equation of elliptic type, which means that the dependence of the solution on the boundary conditions not only in the past but in the future. The question of boundary conditions for equation (1) was discussed by Einstein [1], which came to the conclusion that the universe is a closed spherical world. Another point of view is contained in the monograph [25] in which the author suggested that there are only three types of gravitational fields, and that each type has its own boundary conditions.

Equation (11) can be regarded as a model of the collapse phenomenon spherical rotating body. It is usually assumed that black holes are static formation interacting with the outside world through the gravitational field [26-27]. However, the equation (11) shows that the black hole, if it exists, should be closed not only in space but also in time. Consequently, hole, if it occurred at some point in time as a result of collapse, must have a finite length in time. Indeed, consider the solution of equation (11), depending only on time. Suppose $\psi_{rr} = 0$, then equation (11) takes the form

$$\psi\psi_{tt} = -2(1+b^2)\psi^3 + 2\psi_t^{\ 2} \tag{13}$$

Equation (13) can be integrated in the general case, we find the result

$$\psi(t) = \frac{4(1+b^2)}{4(1+b^2)^2(t-t_0)^2 + C_0} \tag{14}$$

The solution (14) describes the object, localized around a point of time $t = t_0$ as required above.

The field equations of mixed type

Consider the metric of the form

$$\begin{aligned} & ds^2 = \psi(t,r)dt^2 + 2bdtdr - \frac{1+\psi^2}{\psi}dr^2 - d\vartheta^2 - \sin^2\vartheta d\phi^2 \\ & g_{00} = \psi(t,r), \\ & g_{11} = -\frac{1+\psi^2}{\psi}, \\ & g_{01} = g_{10} = b \\ & g_{02} = g_{13} = g_{23} = g_{03} = 0, \\ & g_{22} = -1,\ g_{33} = -\sin^2\vartheta \end{aligned} \tag{15}$$

Nonzero components of the Einstein tensor in the metric (15) are equal to

$$G_{00} = \psi(t,r), G_{01} = G_{10} = b,$$
$$G_{11} = -\frac{1+\psi^2}{\psi},$$
$$G_{22} \sin^2 \vartheta = G_{33},$$
$$G_{22} = \frac{-2(1+b^2)\psi_t^{\ 2} - 3\psi^2\psi_t^2 + \psi^4(\psi_t^2 - \psi_r^2) + \psi^5(\psi_{rr} - \psi_{tt}) + (1+b^2)\psi\psi_{tt} + \psi^3((1+b^2)\psi_{rr} - b^2\psi_{tt})}{2(1+b^2+\psi^2)^2\psi^3}$$

Assuming $G_{22} = -1$, we find the field equation

$$\begin{aligned} &-\psi(\psi^4 + \psi^2 b^2 - b^2 - 1)\psi_{tt} + \psi^3(\psi^2 + b^2 + 1)\psi_{rr} = \\ &-2(1+b^2+\psi^2)^2\psi^3 + (2+2b^2+3\psi^2-\psi^4)\psi_t^{\ 2} + \psi^4\psi_r^2 \end{aligned} \qquad (16)$$

Note that equation (16) changes its type depending on the amplitude of the field ψ:

1) In the region $0 < \psi < 1$ equation (16) is of elliptic type;

2) In the region $\psi > 1$ equation (16) is of hyperbolic type;

3) In the region $\psi = 1$ equation (16) has a parabolic type - see [30].

In all regions signature metric does not change, as seen from expression (15).

In solving the problem of the decay of clusters in the model (16), such as shown in Fig. 1 for a hyperbolic equation (8), an effect that prevents spread of vibrations to the region where eq. (16) has elliptic type - Fig. 2.

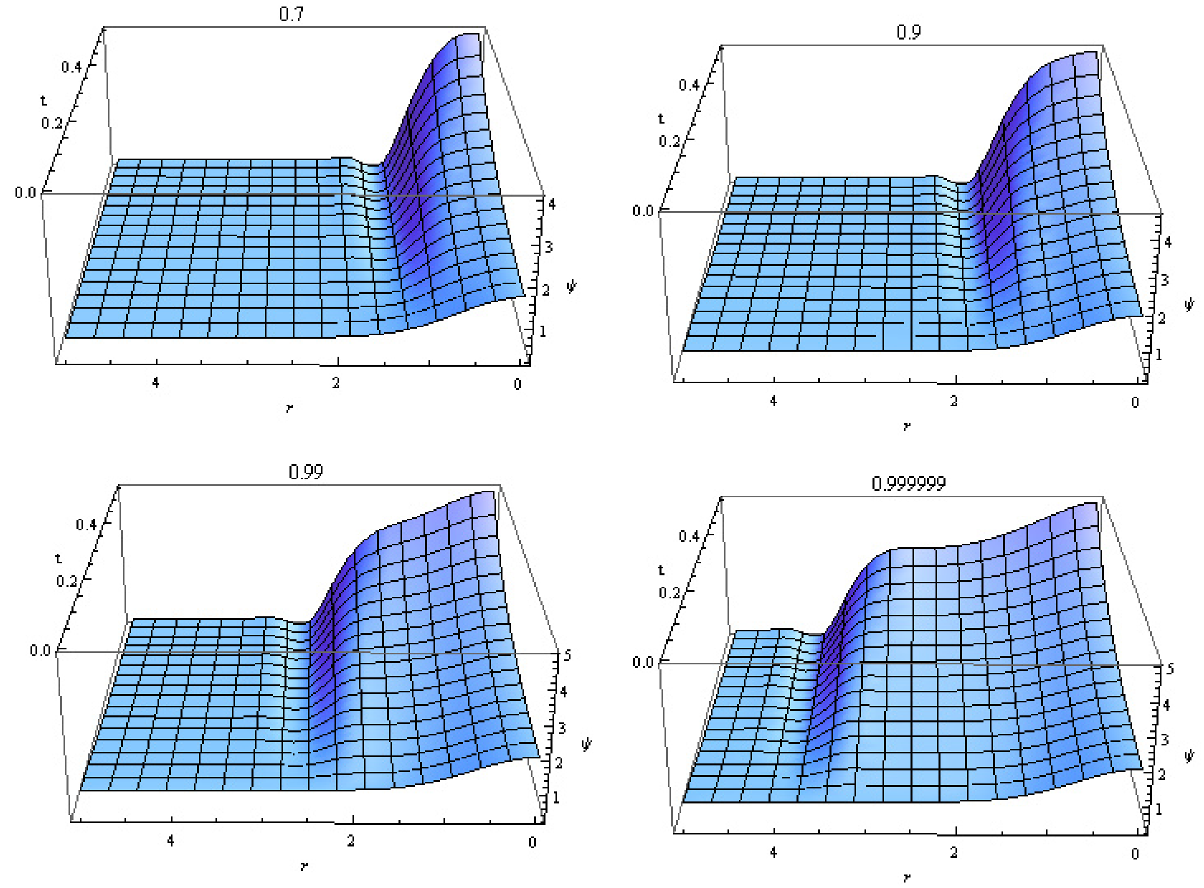

Figure 2: The perturbations of finite amplitude in the model (16). Initial perturbation is given in the form $\psi(0,r)=\psi_0+\exp(-r^2),\ 0<\psi_0<1$. Value of the constant ψ_0 is indicated above each figure.

Specifying the initial distribution of the field in the form of a normal distribution $\psi(0,r)=\psi_0+\exp(-r^2),\ 0<\psi_0<1$, we find that the wave perturbations do not penetrate to the region $0<\psi<1$ and size of this region depends on the parameter ψ_0 - see Fig. 2.

Another effect that is due to the properties of the model (16) - is the decay and merging of finite amplitude waves - Fig. 3.

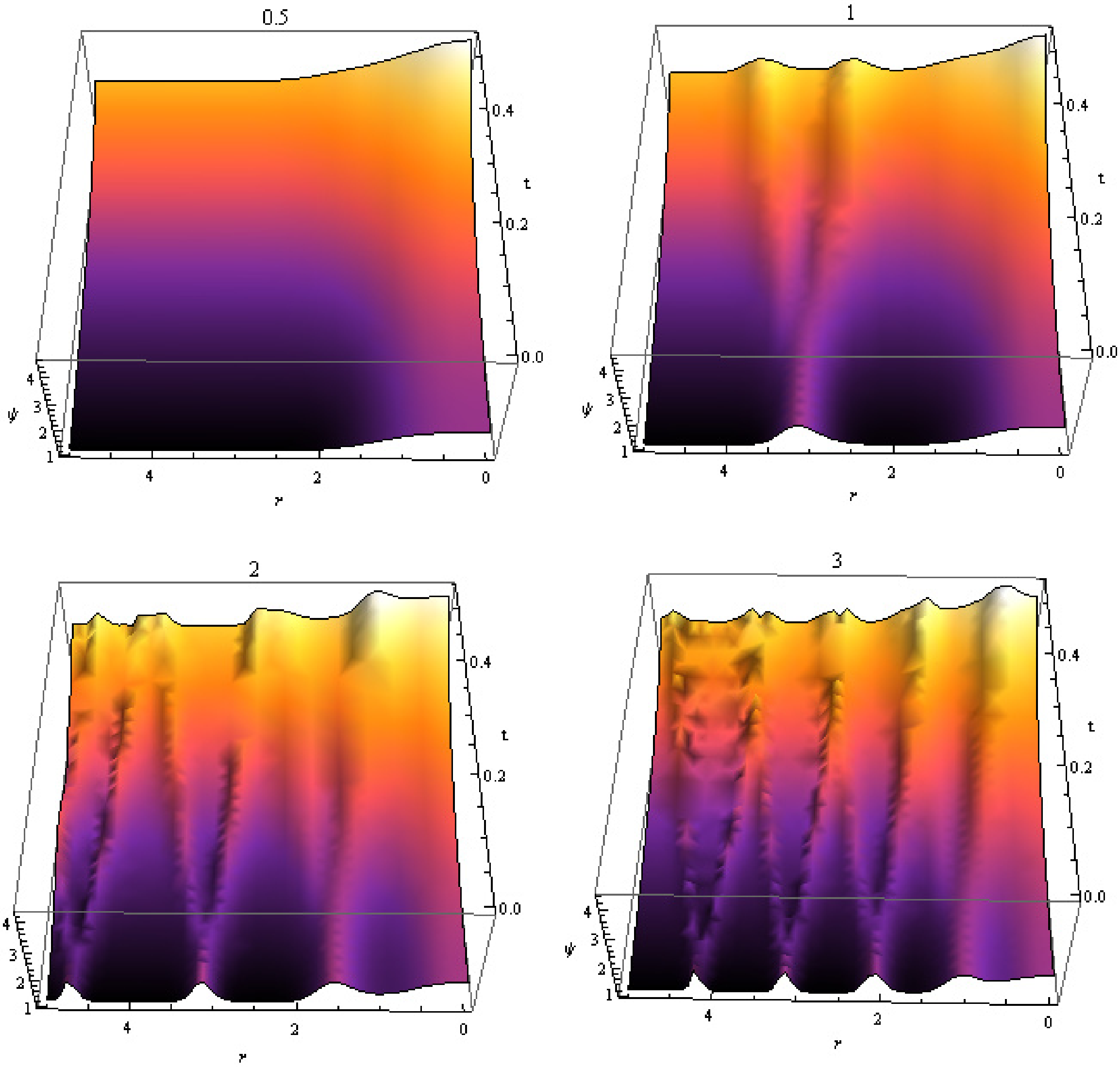

Figure 3: Disintegration of finite amplitude waves in the model (16). Initial perturbation is given in the form $\psi(0,r)=1+\psi_0\exp\left[-r^2\sin(kr)\right], \psi_0>0$. Wavenumber k indicated on the drawings.

If in the hyperbolic model (16) to create a disturbance at the initial moment of finite amplitude, then over time the crest of each wave splits into two waves.

Waves produced by the decay of adjacent ridges merge over time, creating a complex picture of the wave field, typical for nonlinear systems - Fig. 3. This type of processes may be responsible for the formation of the observable matter.

In the region where model (16) has an elliptic type, it shows the main feature of our world arising and disappearing phenomena that every object has a finite lifetime.

Therefore, in general relativity there are gravitational fields that are defined on the whole space - time, but in one area, these fields are defined on the basis of elliptic equations, and in another area - based on hyperbolic equations. As you know, this kind of problems arises in aerodynamics when body moving through the speed of sound, as well as in problems of bending of elastic shells [30]. Note that in equation (16), the transition is not associated with the speed of light, and with the critical amplitude of the gravity potential $\psi = 1$.

In this regard, we note that in the case of the Einstein equations is usually assumed that these equations can be reduced for a particular choice of the metric to the equations of hyperbolic type, so they can formulate the Cauchy problem with initial conditions [24-28, 30-33], which is consistent with the principle of causality. Some authors have suggested that Einstein's field equations are hyperbolic in general [30-31], referring to the known results obtained under certain restrictions by de Donder [32] and Lanczos [33].

However, the metric type (15) allows us to reduce the Einstein equations to equations of mixed type (16), for which we can define the regions of the hyperbolic and elliptic type. The question arises about the origin of these equations and their relation to other phenomena in physics. We assume that the change in the type of the field equations due to a phase transition of dark energy into dark matter and other matter.

The field equations of mixed type and the equation of state

We consider the question of the origin of mixed-type equations in the metric

$$\begin{aligned} &ds^2 = \psi(t,r)dt^2 + 2bdtdr - p(\psi)dr^2 - d\vartheta^2 - \sin^2\vartheta d\phi^2 \\ &g_{00} = \psi(t,r), \\ &g_{11} = -p(\psi), \\ &g_{01} = g_{10} = b \\ &g_{02} = g_{13} = g_{23} = g_{03} = 0, \\ &g_{22} = -1,\ g_{33} = -\sin^2\vartheta \end{aligned} \tag{17}$$

Nonzero components of the Einstein tensor in the metric (17) are equal to

$$\begin{aligned} &G_{00} = \psi(t,r),\ G_{01} = G_{10} = b \\ &G_{11} = -p(\psi), \\ &G_{22}\sin^2\vartheta = G_{33}, \\ &G_{22} = G_{22}(\psi, \psi_t, \psi_r, \psi_{tt}, \psi_{rr}) \end{aligned} \tag{18}$$

Assuming $G_{22} = -1$ we find the field equation

$$-p'\psi_{tt} + \psi_{rr} = -2(b^2 + p\psi) - \frac{pp' - 2(b^2 + p\psi)p'' + p'^2\psi}{2(b^2 + p\psi)}\psi_t^2 + \frac{p + p'\psi}{2(b^2 + p\psi)}\psi_r^2 \tag{19}$$

Note that equation (19) changes its type depending on the sign of the derivative p':

In the region $p' < 0$ equation (19) is of elliptic type;

In the region $p' > 0$ equation (19) is of hyperbolic type;

In the region $p' = 0$ equation (19) has a parabolic type.

Signature of the metric (19) does not change if we require further $p(\psi) > 0$.

In the standard theory of gravity - see, for example, [24-28, 31], communication between the components of the Einstein tensor G_{00}, G_{11} is due to equation of state of the matter, which is the source of the gravitational field. Studied above metrics were constructed based on the model (3), in which matter does not affect the geometry of space - time. However, these results remain valid in the original Einstein's model (1), in which the energy-momentum tensor of matter coincides with the metric tensor up to a constant factor.

Therefore, the alleged connection between the components G_{00}, G_{11} in the form $p = p(\psi) > 0$ means that there is the equation of state of some matter or energy that produces a gravitational field. In modern astrophysics energy of this type is called dark energy. It is believed that dark energy is the ground state of the universe. Assuming that there is an equation of state, we thus endow dark energy specific properties that allow comparing the dark energy to other forms of energy and matter.

For ordinary matter change in the sign of the derivative p' indicates the presence of a phase transition. Consequently, if the dark energy can make a phase transition than in this case arise above region of hyperbolic and elliptic type of field equations. One would assume that $p' > 0$ for all values of the argument, and hence equation (19) is of hyperbolic type in the whole space - time. But this would mean that we attach some properties to dark energy, which it could not have.

Gravitational geon

Note, the sign of the function $p = p(\psi)$ does not affect the type of the field equations (19), but, of course, affect its decision and topology. As noted by the authors [26], if all the space-time is globally hyperbolic, it has a very boring topology. On the other hand, if we assume that the signature of the metric is changed, then the space - time has a complex topology. Indeed, consider the static solutions. In this case equation (19) takes the form

$$\psi_{rr} = -2(b^2 + p\psi) + \frac{p + p'\psi}{2(b^2 + p\psi)}\psi_r^2 \qquad (20)$$

Integrating (20), we find

$$(b^2 + p\psi)(C - 4\psi) = \psi_r^2 \tag{21}$$

From here C is an arbitrary constant. For any given function $\psi = \psi(r)$, equation (21) allows to define a function $p = p(r)$, and thus defined in parametric form the equation of state $p = p(\psi)$. For example, the gravitational field of a point mass in the metric (17) corresponds to the solution

$$\psi = -2mr^3 + r^2 \tag{22}$$

Here m - arbitrary constant. Reduce (17) to the form spherically symmetric metric. To do this, multiply the first equation (17) $1/r^2$ to make the substitution $r \to 1/r$ and using (22) we obtain

$$\begin{aligned} &r^2 ds^2 = r^2\psi(1/r)dt^2 - p(\psi)dr^2/r^2 - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta d\phi^2 \\ &r^2\psi(1/r) = 1 - \frac{2m}{r} \end{aligned} \tag{23}$$

This last equation (23) allows us to determine the constants in the solution (22). Using the solution (22) we can determine the equation of state, which corresponds to the gravitational field of a point mass and the parameters of the metric (23) – see Fig. 4.

Schwarzschild solution [34] is typically used for evidence of gravitational collapse of a spherical body [24-28, 31]. Collapse of stars and black holes form both are a significant part of modern research in astrophysics. It is believed that during the collapse of the matter is irreversibly lost inside a black hole. However, this phenomenon can be seen from another point of view, as the mechanism of generation the observed matter from dark energy. Indeed, the metric (23) depends on the equation of state of dark energy (21). In turn, this equation relies on two constants that allow reproducing particles having gravity mass. But by virtue of the principle of equivalence such a point mass has inertia. Therefore, a black hole can be regarded as an elementary particle of mass m .

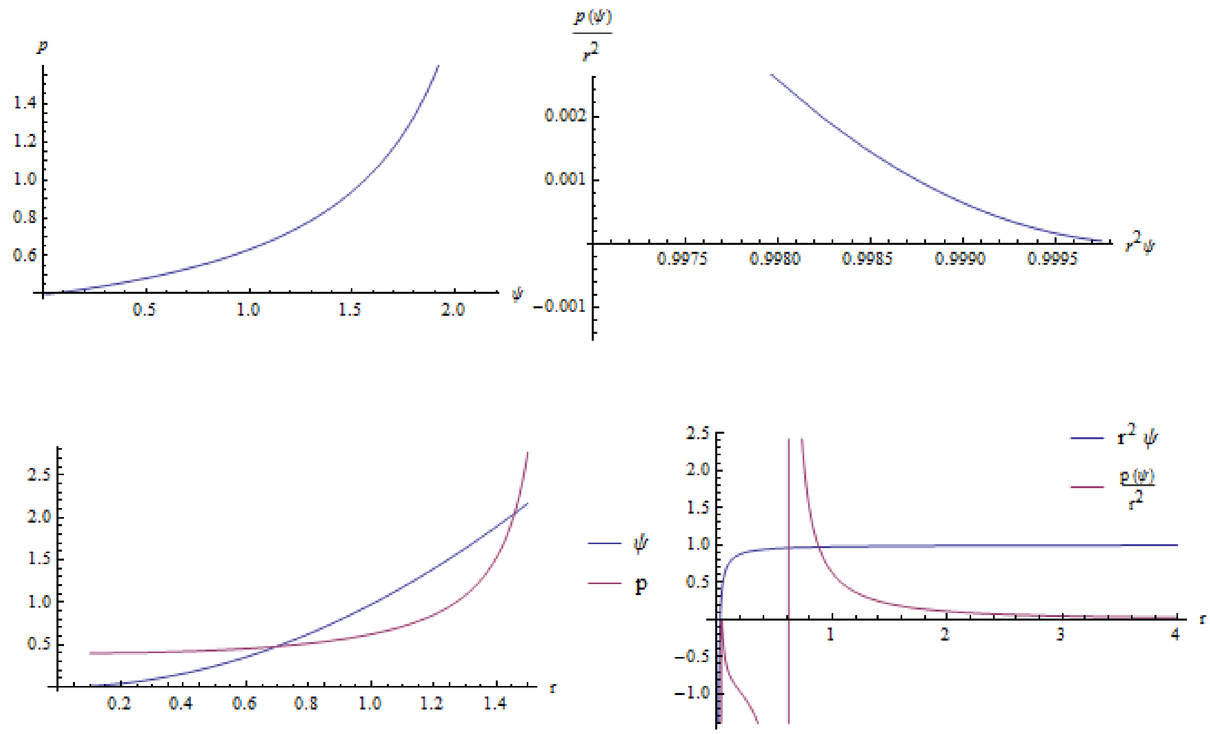

Figure 4: Equation of state and parameters of the metric (23) - on the left, and the metric (17) - on the right, which correspond to the field of a point mass: $C = 10; m = 0.0125; b = 0$.

The existence of such a particle is first pointed Wheeler [6], who gave name geon or mass without mass [8]. Wheeler [6-8] suggested that geon formed by the gravitational interaction of electromagnetic fields. Later it was shown that solutions describing geon exist in the theory of gravitation [35] and in quantum gravity [36].

Unsteady galactic field

Given that in our part of the universe and galaxy clusters are removed from each other by the Hubble law, which is a consequence of the quadratic potential (12), we can assume that this whole area of space-time is an area in which the field equations have elliptic type. But in this case the local gravitational field should not only depend on the coordinates, but from time too, and this dependence is determined by solutions of elliptic equation (19). In the case of the metric in the galaxy scale equation (19) can be greatly simplified by replacing it with a linear equation. Introduce new coordinates and time formulas

$$t \to t\sqrt{\frac{2p}{-p'}}, \quad r \to r\sqrt{2p}, \, p' < 0$$

Then in the case of $b = 0$ equation (19) can be written as

$$\psi_{tt} + \psi_{rr} = -\psi \tag{24}$$

Note that in this approximation, the superposition principle holds. For equation (24) there are the following solutions, joint with the idea of the origin and evolution of galaxies

$$\begin{aligned} \psi &= g\cos(mr)\sin(t\sqrt{1-m^2}), \quad m<1 \\ \psi &= g\cos(mr)\sinh(t\sqrt{m^2-1}), \quad m>1 \end{aligned} \tag{25}$$

We assume that the gravitational potentials of the type (25) may be associated with the formation of galactic nuclei and jets. As an example, consider the motion of stars in the nonrelativistic approximation in the gravitational field with potential (25). The equation of motion in this approximation has the form

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{1}{2}\nabla\psi = \begin{cases} \frac{1}{2}mg\sin(t\sqrt{1-m^2})\sin(mr)\frac{\mathbf{r}}{r}, & m<1 \\ \frac{1}{2}mg\sinh(t\sqrt{m^2-1})\sin(mr)\frac{\mathbf{r}}{r}, & m>1 \end{cases} \tag{26}$$

Fig. 5-6 shows the trajectory of motion of test bodies in the case $g = -2\cdot 10^2$. For a periodic potential at $1-m^2 \to 0, m<1$, the trajectory begins at the periphery of the system, the particle performs multiple vibrations toward the center. In this case, a structure similar to the two-arm galaxy, but with a decrease of attraction movement occurs on the periphery of the system, thereby forming a halo around the original structure - the right and left figures 5 respectively.

Figure 6 presented trajectories of test particles in a monotonically increasing potential at $m^2 - 1 \to 0, m > 1$. In this case also, a structure similar to the two-arm galaxy and halo form particles that move at the periphery of the system - left and right figures 6, respectively.

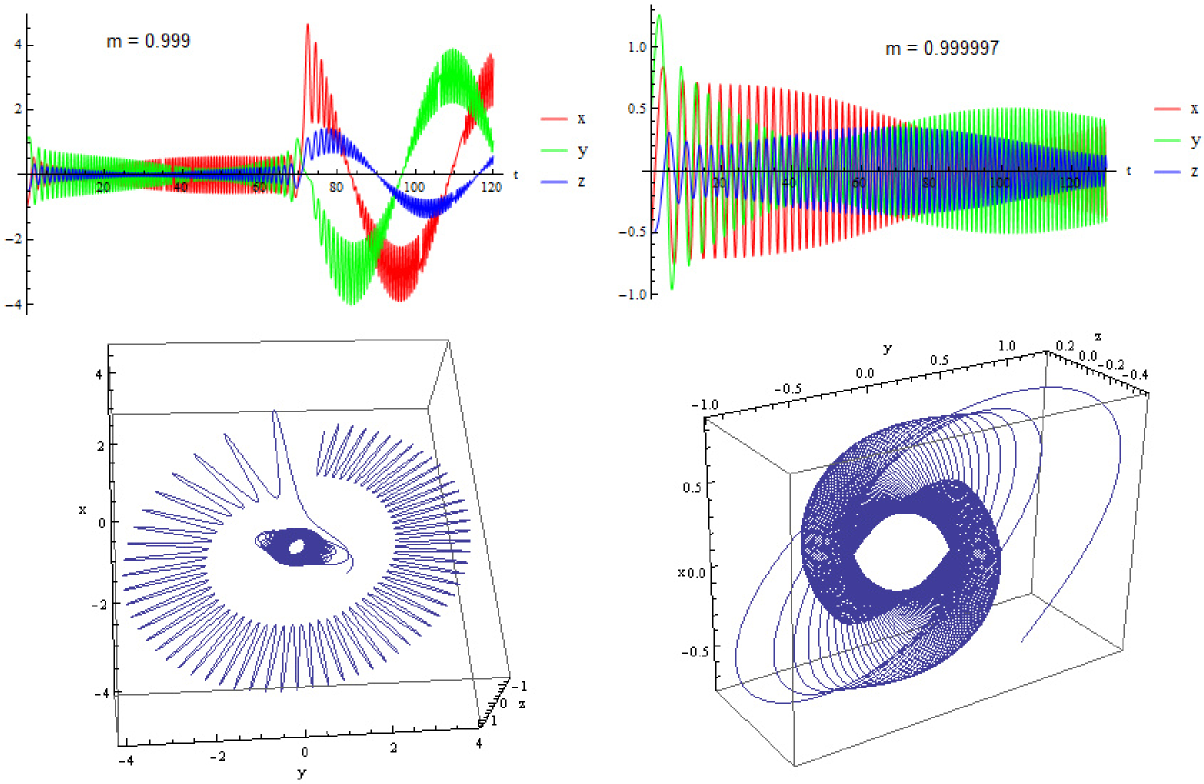

Figure 5: The trajectory of a test particle in a gravitational field (25) at $1 - m^2 \to 0, m < 1$

Consider the class of gravitational fields defined in a circle in the polar coordinate system

$$\rho = \sqrt{t^2 + r^2}, \quad \sin\chi = t / \rho \tag{27}$$

$$\frac{\partial^2\psi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\psi}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\chi^2}=-\psi \quad (28)$$

For equation (28) we can find exact solutions of the cylindrical Bessel functions. We assume that the gravitational potentials of this type may also be associated with the formation of nuclei and galactic jets. In addition, based on equation (28) can be carried out simulations of the effect of an event on the metric system. These include, for example, the decays of elementary particles.

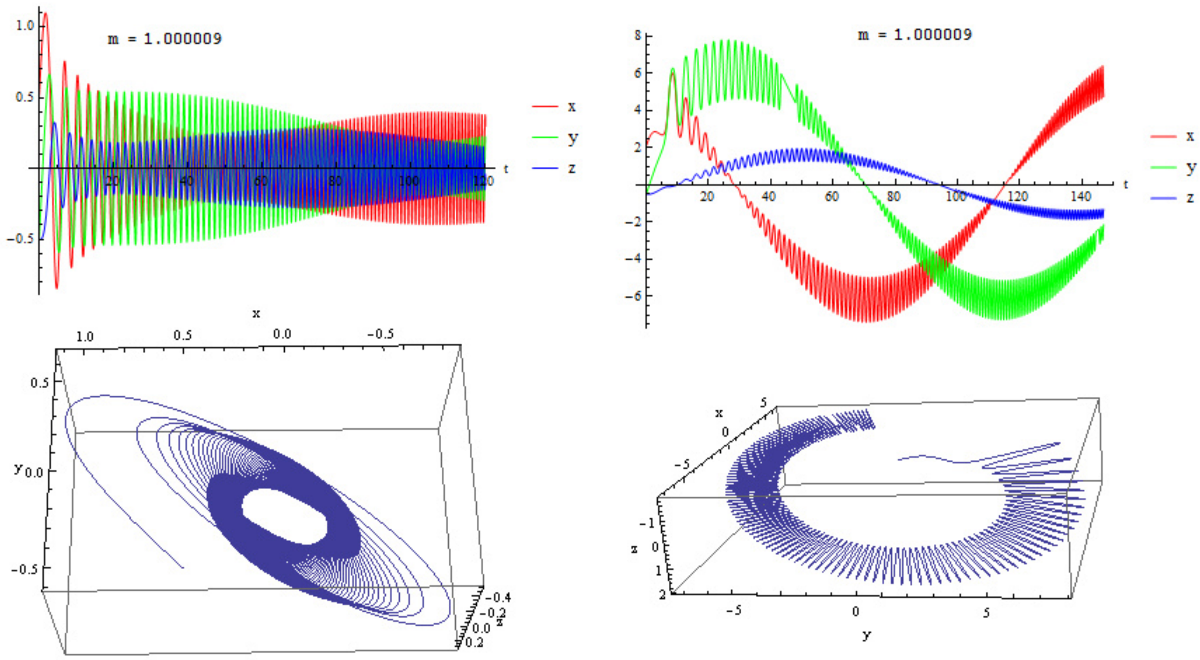

Figure 6: The trajectory of a test particle in a gravitational field (25) at $m^2-1\to 0, m>1$

Finally, we note that for any kind of gravitational potential set of spiral galaxies on the basis of experimental data [37], we can specify the corresponding solution of (24), dependent on time.

Hadrons metric model

Consider a centrally symmetric metric of the form [38]

$$\Psi = \eta_{ij}\omega^i\omega^j = -dt^2 + e^{2\nu}dr^2 + d\theta^2 + \sigma^2(\theta)d\varphi^2$$
$$\frac{d^2\sigma}{d\theta^2} = -\kappa\sigma \qquad (29)$$

Here $\eta_{ij} = \eta^{ij}$ - the metric tensor of Minkowski space of signature (- + + +), $\kappa = const$ - Gaussian curvature of a quadratic form $d\theta^2 + \sigma^2(\theta)d\varphi^2$, function $\nu = \nu(r,t)$ is determined by solving the Yang-Mills equations [38]. Among all the solutions of the Yang -Mills equations, in the case of the metric (29), there is one that is expressed in terms of the Weierstrass elliptic function [38]. In this case, the model equations reduce to the form [39-43]:

$$A_{\tau\tau} = \frac{1}{2}(A^2 - \kappa^2),\ e^{\nu} = A_{\tau}, \quad \tau = t \pm r + \tau_0$$
$$A = \sqrt[3]{12}\wp(\tau/\sqrt[3]{12}; g_2, g_3), \qquad (30)$$
$$b_{11} = -b_{22} = \frac{1}{3}A - \frac{\kappa}{6},\ b_{33} = b_{44} = \frac{1}{6}A - \frac{\kappa}{3},\ b_{12} = b_{21} = 0.$$

Here indicated: g_2, g_3 - invariants of the Weierstrass function, and $g_2 = \kappa^2\sqrt[3]{12}$, τ_0 - free parameter related to the choice of origin, $b_{ij} + b_{ji} - 2(\eta^{ij}b_{ij})\eta_{ij} = T_{ij}$ - energy-momentum tensor of matter. Note that in this notation, the Einstein equation (1) has the form

$$b_{ij} + b_{ji} + b\eta_{ij} = R_{ij}$$

$b = \eta^{ij}b_{ij}$; R_{ij} - Ricci tensor.

Metric (29-30) was used to simulate the dynamics of quarks and hadrons composed of atomic nuclei as well as preons composed of leptons and quarks [39-43]. We show that a similar metric, depending on the Weierstrass function, contained among the solutions of equation (21). To do this we assume that the state equation can be written as

$$p(\psi) = -\psi + \frac{b^2}{\psi} + \frac{g_2}{4\psi} + \frac{g_3}{4\psi^2} \tag{31}$$

Then, in case $C = 0$ the solution of equation (21) has the form

$$\psi = \wp(r - r_0, g_2, g_3) \tag{32}$$

Thus, it was found that the periods of the Weierstrass function in the metric (32) are associated with the equation of state of dark energy. Note that the metric depends on the Weierstrass elliptic function, first pointed out by Delsarte [18].

Invariants of the Weierstrass function in the metric (30) may coincide with those of the metric (32). This coincidence means that the Yang-Mills equations describe the behavior of dark energy equation of state with a given type (31) under certain restrictions imposed by the gauge symmetry.

Indeed, the constant is a solution of the first equation (30), but this decision in the case of a constant equal to unity corresponds to the execution of the equation (19). Hence, the difference in the modeling of the metric in terms of the model (3) and in Yang-Mills theory is the choice of the constants g_2, g_3, which in the case of Yang-Mills theory are arbitrary, and Einstein's theory depends on the equation of state.

Consider the motion of particles in a gravitational potential (32) in the nonrelativistic approximation. Using the equation of type (26), we find that in this case there are trajectories that depend not only on the coupling constant and the initial data, but also on the invariants of the Weierstrass function - Fig. 7. Note that some cases of trajectories in a conformal space, depending on the Weierstrass elliptic function, obtained in [44].

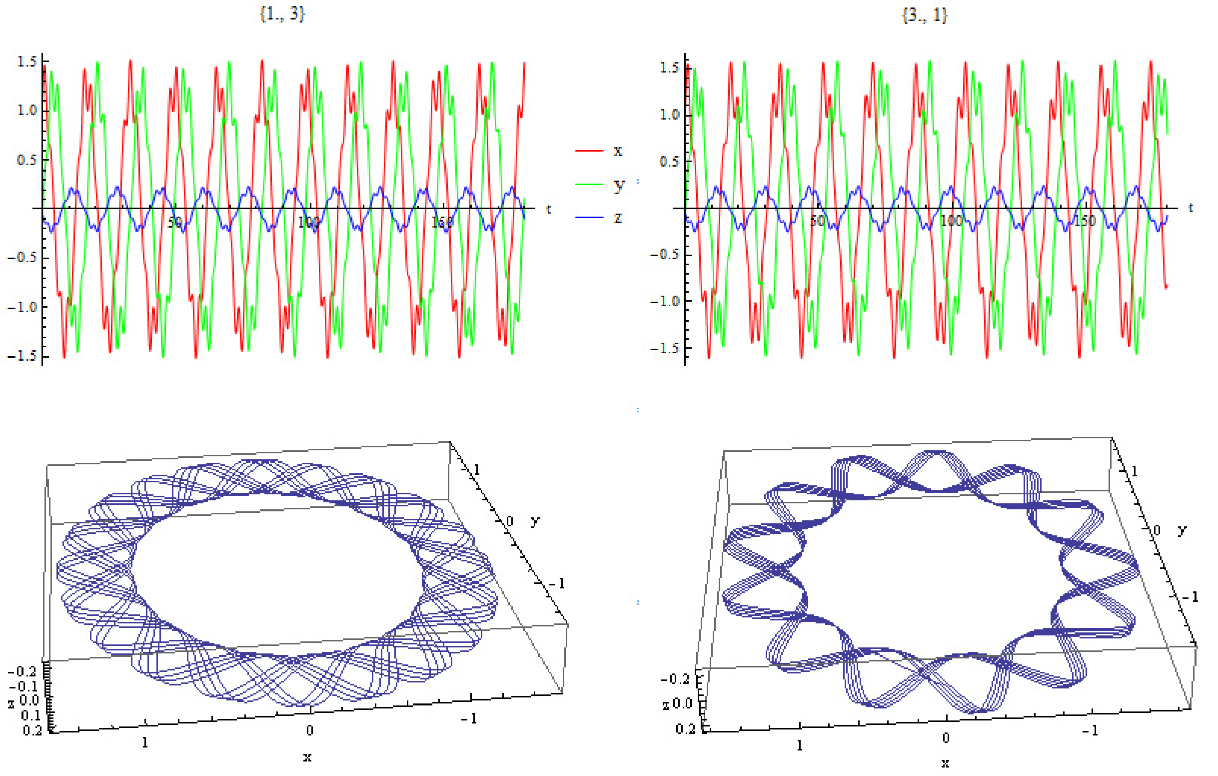

Figure 7: The trajectory of a test particle in a gravitational field (32). Above figures are invariants of the Weierstrass function g_2, g_3.

It is known that Yang-Mills theory is the basic model for quantum chromodynamics, and Einstein's theory is the basic model for a quantum theory of gravity [7-8]. In this and in the other case there is a problem of quantization due to the nonlinearity of the models [45]. The above theory allows us to combine the Yang-Mills theory and Einstein's theory in solving the problem of quantization. This kind of association may be based on the metric (17) with the field equations of mixed type and the equation of state, permitting and singular solutions of the type of geon, and solutions of the type (32). This approach allows capturing the diversity of matter, which produces nature.

Finally, we note that the theory of fields with mixed type equation (19) can be compared with the theory [46], which are imposed non-commutating variables, allowing distant parts of the universe instantaneously communicate with each other. In the case of the field equations of elliptic type this interaction means that the metric is defined on the whole time interval of the existence of the material universe. However, consideration of these issues is beyond the scope of this paper.

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