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ДИНАМИКА КВАРКОВ В АТОМНЫХ ЯДРАХ И КВАРКОВЫЕ ОБОЛОЧКИ

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В работе рассмотрена система уравнений Дирака, описывающая динамику кварков в метрике атомных ядер. Показано, что энергия связи нуклонов для всех известных нуклидов может быть представлена как функция от содержания кварков. Полученные зависимости энергии связи нуклонов свидетельствуют о наличии кварковых оболочек, аналогичных электронным оболочкам

Ключевые слова: КВАРКИ, НЕЙТРОН, МАГНИТНЫЙ МОМЕНТ, МЕТРИКА, ПРОТОН, ЭНЕРГИЯ СВЯЗИ, ЯДРО

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QUARK DYNAMICS IN ATOMIC NUCLEI AND QUARK SHELLS

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In this paper we consider a system of Dirac equations describing the dynamics of quarks in the metric of the atomic nuclei. We found out, that the binding energy of the nucleons for all known nuclides depends on the content of the quarks. The resulting dependence of the energy of the nucleons shows a quark shells, similar electron shells

Keywords: BINDING ENERGY, QUARKS, MAGNETIC MOMENT, METRIC, PROTON, NEUTRON, NUCLEI

Introduction

According to the theory of nuclear shells [1-4], the periodic patterns in the nuclei are explained by analogy with the electron shells, the Pauli Exclusion Principle, which is applied separately for protons and neutrons filling the nuclear envelope. On the other hand, the models of quantum chromodynamics, which are widely used for modeling of hadrons and nuclei [3-5], the nucleons, are presented as composite particles composed of quarks. The question arises as to whether any of the quark shells in nuclei, similar electron shells? In [7-9] formulated a model of hadrons metric satisfying the basic requirements of particle physics and cosmology, as well as the dynamics of quarks interacting with a Yang-Mills field. Results are obtained on the magnetic moments of baryons, in agreement with the experimental data with high accuracy. In this paper we consider the application of the model of the dynamics of quarks [8-9] to the modeling of the energy of the nucleons in nuclei. The equation of energy of the nucleons as a function of the content of quarks is proposed. It is shown that the quarks form shells in nuclei similar electron shells in atoms.

The basic equations of the model metrics hadrons

Consider a centrally symmetric metric of the form [7-10]

$$\Psi = h_{ij} w^i w^j = -dt^2 + e^{2n} dr^2 + dq^2 + s^2(q) dj^2$$
$$\frac{d^2 s}{dq^2} = -ks \tag{1}$$

$$w^1 = dt, w^2 = e^n dr, w^3 = dq, w^4 = s\,dj$$

Here $h_{ij} = h^{ij}$ - is the metric tensor of the Minkowski space of signature (- + + +), $k = const$ - Gaussian curvature of the quadratic form $dq^2 + s^2(q)dj^2$, $n = n(r,t)$ - function is determined by solving the Yang-Mills equations [7-10]. Wherever not specified, we use the system of units with speed of light=Plank constant =1.

Among all the solutions of the Yang-Mills equations, in the case of the metric (1), there is one which is expressed in terms of Weierstrass elliptic function [10]. In this case, the model equations reduced to the form [7-9]:

$$A_{tt} = \frac{1}{2}(A^2 - k^2), e^n = A_t, \quad t = t \pm r + t_0$$
$$A = \sqrt[3]{12}\wp(t / \sqrt[3]{12}; g_2, g_3), \tag{2}$$
$$b_{11} = -b_{22} = \frac{1}{3}A - \frac{k}{6}, b_{33} = b_{44} = \frac{1}{6}A - \frac{k}{3}, b_{12} = b_{21} = 0.$$

It is indicated: g_2, g_3 - invariants of the Weierstrass function, and $g_2 = k^2\sqrt[3]{12}$; t_0 - free parameter related to the choice of origin, $b_{ij} + b_{ji} - 2(h^{ij}b_{ij})h_{ij} = T_{ij}$ - energy-momentum tensor of matter. Note that in this notation, the Einstein equations have the form

$$b_{ij} + b_{ji} + bh_{ij} = R_{ij} \tag{3}$$

$b = h^{ij}b_{ij}$; R_{ij} - Ricci tensor.

In the metric (2) can be defined lattice defect such as a bubble. In the bubble we put $A^2 = k^2$, while the solution in the outer region given in the form (2), therefore we have

$$\begin{aligned} &A^2 = k^2, e^n = 0, \quad |t| < t_0 \\ &A = \sqrt[3]{12}\wp(t / \sqrt[3]{12}, g_1, g_2), e^n = A_t, |t| > t_0 \end{aligned} \tag{4}$$

On the borders of the bladder function A and its first derivative is continuous,

$$k = \sqrt[3]{12}\wp(t_0 / \sqrt[3]{12}, g_1, g_2), \ A_t = 0, |t| = t_0 \tag{5}$$

In the particular case of a lattice with the invariants given in the form $g_2 = \sqrt[3]{12}$, $g_3 = 1$, we find the first zero and the corresponding parameter of the metric as $t_0 = 3.0449983$, $k = 2.1038034$. Note that the metric in the interior of the bladder is a three-dimensional one, because it does not contain the radial coordinate. Indeed, using equation (1) and (4), we find

$$\Psi = -dt^2 + dq^2 + \cos^2(\sqrt{k}q + q_0)dj^2 \tag{6}$$

Similarly, the solution is constructed for the other roots of the second equation (5). All of these solutions differ by only the size of the bubble, whereas the value k does not change.

Any bubble can be turned inside out, just reversed inequality (4). In this case, you can define a metric in the outer region of bubble, using the solution of the first equation (2), so that the external space metric coincides with the metric of the Universe [7]. Finally, the third type of particles can be formed as a combination of the first two, and the result is a bubble, a restricted shell of finite thickness [7-9].

Let transform the metric (6) to standard form. To do this, multiply both sides of (6) on a constant $-k$ and introduce new variables that differ from the old variables by a constant factor $\sqrt{k}$, as a result we find

$$\Psi \to \Psi_1 = dt^2 - dq^2 - \sin^2 q dj^2 \qquad (7)$$

The metric (7) was used to model the structure of baryons, including the proton and neutron [8-9].

Dynamics of quarks

To simulate the dynamics of quarks in the interior of the bubble with a metric of the form (7) we consider the system of Dirac equations in an external Yang-Mills field. Note that according to (2) in the metric (7) energy-momentum tensor is constant. Therefore, we assume that the Yang-Mills field in the interior of the bubble is reduced to a set of constants. This model uses three constants, and the field itself is described by a scalar and vector potential

$$B_m^b = (f^b, A_m^b) .$$

In addition, we consider the electromagnetic field, which generated by quarks. Using the results from [11], we transform the Dirac equation to the curvilinear coordinates (7). Therefore we have system of equations

$$ig^{m}(\nabla_{m} + iq_{ab}A^{b}_{m})y_{a} = m_{ab}y_{a} \qquad (8)$$

Here indicated $g^{m}, q_{ab}, A^{b}_{m}, y_{a}, m_{ab}$ - Dirac matrices, the interaction parameters, the vector potential, the wave function and the effective mass of the quark *a* as part of the particle *b*, respectively. Dirac matrices in the metric (7) have the form

$$g^{0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad g^{j} = \begin{pmatrix} 0 & 0 & 0 & -ie^{-ij} \\ 0 & 0 & ie^{ij} & 0 \\ 0 & ie^{-ij} & 0 & 0 \\ -ie^{ij} & 0 & 0 & 0 \end{pmatrix}$$

$$g^{q} = \begin{pmatrix} 0 & 0 & -\sin q & e^{-ij}\cos q \\ 0 & 0 & e^{ij}\cos q & \sin q \\ \sin q & -e^{-ij}\cos q & 0 & 0 \\ -e^{ij}\cos q & -\sin q & 0 & 0 \end{pmatrix}$$

In this notation, the Dirac operator in the metric (7) can be written as

$$g^{m}\nabla_{m} = g^{0}\partial_{t} + g^{q}\partial_{q} + \frac{g^{j}}{\sin q}\partial_{j} \, .$$

Since quarks have an electric charge, they generate an electromagnetic field through which interact with each other. To describe this interaction using the equations of quantum electrodynamics in the form of

$$aq_{ab}\bar{y}_{a}g^{m}y_{a} = (\partial_{t}^{2} - \nabla^{2})A^{m}_{e} \qquad (9)$$

Here $a = e^2 / \mathbf{h}c$ is the fine structure constant; $\overline{y}_a = y_a^+ g^0, y_a^+$ - conjugated (by Hermite) vector. Thus, we assume that the currents and charges of the quarks are added, creating a collective field that quarks interact in accordance with equations (8).

The system of equations (8) - (9) was used to model the dynamics of quarks in the case of baryons [8]. In the simplest case, which takes into account only one electromagnetic field, the model contains 15 non-linear partial differential equations. To reduce the order of the system we represent the solution of equations (8) - (9) in the form

$$y_a = e^{-iwt+iLj} \begin{pmatrix} f_1(q) \\ f_2(q)e^{ij} \\ if_3(q) \\ if_4(q)e^{ij} \end{pmatrix}_a \tag{10}$$

Here L, w are the projection of the angular momentum on the selected axis and the energy of system, respectively. The system of Dirac equations for the case of the representation of the solution in the form (10), reduced to a form

$$\begin{gathered} f_1' = (L + q_{ab}A_b \sin q)(f_1 \cot q + f_2) + f_2 + \\ (m_{ab} + w - q_{ab}\Phi_b)(f_3 \sin q - f_4 \cos q) \\ f_2' = (L + q_{ab}A_b \sin q)(f_1 - f_2 \cot q) - f_2 \cot q - \\ (m_{ab} + w - q_{ab}\Phi_b)(f_3 \cos q + f_4 \sin q) \\ f_3' = (m_{ab} - w + q_{ab}\Phi_b)(f_1 \sin q - f_2 \cos q) + \\ (L + q_{ab}A_b \sin q)(f_3 \cot q + f_4) + f_4 \\ f_4' = -(m_{ab} - w + q_{ab}\Phi_b)(f_1 \cos q + f_2 \sin q) + \\ (L + q_{ab}A_b \sin q)(f_3 - f_4 \cot q) - f_4 \cot q \end{gathered} \tag{11}$$

Here we assume that $A_b = A_e + A_{YM}, \Phi_b = \Phi_e + \Phi_{YM}$.

Note that the mass and charge are individual for each quark, and the angular momentum and energy of the system are chosen to make standing waves. Calculating the current in the left-hand side of equation (9) and the nabla operator in the right-hand side, we find the equations describing the electrodynamics part of the potential

$$a q_{ab} \overline{y}_a g^0 y_a = a q_{ab} \left(\sum_{i=1}^{4} f_i^2 \right)_a = -\Phi_e'' - \Phi_e' \cot q \ , \qquad (12)$$

$$a q_{ab} \overline{y}_a g^j y_a = 2 a q_{ab} \left(f_1 f_4 - f_2 f_3 \right)_a = -A_e'' - A_e' \cot q + \frac{A_e}{\sin^2 q} \ ,$$

$$\overline{y}_a g^q y_a = 0 \ .$$

Here, with the index a there is a summation of all the quarks in the system. Thus, in the case of nucleons problem is reduced to solving a system of 14 ordinary differential equations.

As is known, the electromagnetic properties of elementary particles are characterized by electric charge and magnetic moment. Therefore, the parameters of the Yang-Mills field, appearing in (11), should be related to the charge and magnetic moment of the quarks system, which are defined as follows

$$Q_b = \int dV q_{ab} \overline{y}_a g^0 y_a = 4p \int_0^{p/2} dq \sin q \, q_{ab} \left(\sum_{i=1}^{4} f_i^2 \right)_a \qquad (13)$$

$$m_b = \frac{1}{2}\int dV[\mathbf{r}\times\mathbf{j}]_z = 2pm_q \int_0^{p/2} dq \sin^2 q\, q_{ab}\overline{y}_a g^j y_a =$$

$$4pm_q \int_0^{p/2} dq \sin^2 q \sum_a q_{ab}\left(f_1 f_4 - f_2 f_3\right)_a$$

The unit of measurement of mass let take 1 MeV, then the parameters of the Yang-Mills field, the vector potential and the energy of the system will be expressed in units of MeV. The unit of the magnetic moment in this case is $m_q = 1.0219978\, m_B$, where the Bohr magneton is m_B , and a numerical factor equal to two electron mass, expressed in the unit of MeV.

Model of nucleon

Vector potential impact on the parameters of baryons investigated in [8]. It was found that the scale of variation of the vector part of Yang-Mills field is less than 1 MeV. Therefore, we can exclude it by replacing with a scalar potential, which impacts the effective mass of the quarks [9, 12]. The solution of the system of equations (11) - (12) with zero vector potential of the Yang-Mills field can be obtained as a series in powers of the parameter a . For a system of quarks ground state with zero angular momentum is in the standard form (10) with constant functions f_i:

$$L = 0,\ f_1 = f_{ab},\ f_2 = 0,\ f_3 = f_4 = g_{ab} \qquad (14)$$

In the case of (14) the system (11) with zero vector potential reduces to:

$$2g_{ab} + (m_{ab} - w_{ab}) f_{ab} = 0,\ w_{ab} = -m_{ab} \qquad (15)$$

Calculating the components of the 4-vector current, and using the first normalization condition (13), we have finally

$$\begin{aligned} j^0 &= f_{ab}^2 + g_{ab}^2 = (1+m_{ab}^2)\, f_{ab}^2\,, \\ j^j &= 2\, f_{ab}\, g_{ab} \sin q = -2 m_{ab}\, f_{ab}^2 \sin q\,, \\ 4pj^0 &= 1,\ f_{ab}^2 = \frac{1}{4p\,(1+m_{ab}^2)} \end{aligned} \tag{16}$$

Let use these results to calculate the magnetic moments of the neutron and proton. General properties of nucleons and quarks are presented in Tables 1-2.

Table 1: Properties of baryons

Symbol	Spin	Charge	Mass	BaryonNumber	GFactor	Hypercharge	Isospin	QuarkContent
p	$\frac{1}{2}$	1	938.27203	1	5.585694713	1	$\frac{1}{2}$	{{DownQuark, UpQuark, UpQuark}}
$\bar{p}$	$\frac{1}{2}$	−1	938.27203	−1	5.585694713	−1	$\frac{1}{2}$	{{DownQuarkBar, UpQuarkBar, UpQuarkBar}}
n	$\frac{1}{2}$	0	939.56536	1	−3.82608545	1	$\frac{1}{2}$	{{DownQuark, DownQuark, UpQuark}}
$\bar{n}$	$\frac{1}{2}$	0	939.56536	−1	−3.82608545	−1	$\frac{1}{2}$	{{DownQuarkBar, DownQuarkBar, UpQuarkBar}}

Table 2: Properties of quarks

Symbol	Spin	Charge	Mass	BaryonNumber	Bottomness	Charm	Hypercharge	Isospin	Strangeness	Topness
u	$\frac{1}{2}$	$\frac{2}{3}$	2.2	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{2}$	0	0
$\bar{u}$	$\frac{1}{2}$	$-\frac{2}{3}$	2.2	$-\frac{1}{3}$	0	0	$-\frac{1}{3}$	$\frac{1}{2}$	0	0
d	$\frac{1}{2}$	$-\frac{1}{3}$	5.0	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{2}$	0	0
$\bar{d}$	$\frac{1}{2}$	$\frac{1}{3}$	5.0	$-\frac{1}{3}$	0	0	$-\frac{1}{3}$	$\frac{1}{2}$	0	0

If we assume that the quarks of type u in the proton have opposite spins, and in the neutron quarks d have opposite spins, while the magnetic moment of the proton depends on the effective mass of the quark d, and the neutron magnetic moment depends on the effective mass of the quark u. Under these assumptions, we find

$$m_b / m_q = -\frac{2m_{ab}q_{ab}}{3(1+m_{ab}^2)}, b = n, p; a = u, d. \tag{17}$$

In the case of the proton we have $m_p / m_q = 1.5544916 \times 10^{-3}$, respectively, equation (17) has two roots

$$m_{dp} = 0.00699556\,MeV; 142.948\,MeV\ . \tag{18}$$

For the neutron $m_n / m_q = -1.06479466 \times 10^{-3}$, and cosicvently for the effective mass of the quark u we have two roots:

$$m_{un} = 0.0023958\,MeV; 417.397\,MeV\ . \tag{19}$$

Consequently, in each case, we have two roots of the equation (17). One of them is corresponding to the very low energy of the quarks of a few keV. The second root (18) is close to the mass of the charged pi-meson - 139.57018 MeV, and the second root of (19) is close to the value of three pi-meson mass.

Modeling the binding energy of the nucleons in nuclei

As it well known, the nucleons combined together in atomic nuclei under the influence of the nuclear forces. However, the nuclear forces themselves have long remained a mystery, despite numerous phenomenological models that simulated hypothetical nuclear forces like as the Yukawa potential, the Woods-Saxon potential or the quantum harmonic oscillator provides the base for the nuclear shell model [1]. Notable progress in the modeling of nuclear forces associated with the development of quantum chromodynamics [13-14] and numerical models of nucleons and light nuclei [5-6].

In [9] we have developed a model that allows us to explain the nature of the nuclear forces of the dynamics of quarks in the metric (7). It is assumed that the nucleus consists of shells with a metric of the type (7) and the gluon condensate [15-18]. Consider the application of the model [9] to modeling the energy of the nucleons in nuclei.

Our basic assumption is that each nucleon in the nucleus loses its individuality by dissociation to individual quarks that form quark shells. These shells are filled sequentially, just as filled electron shells. Since the nucleons are composed of two types of quarks, there are two types of shells that are filled with u and d quarks, respectively. In this case, the binding energy per nucleon depends on the concentration of quarks in the shells and the energy of the interaction of quarks.

We can also assume that the quarks of each type form a Fermi gas, which has the chemical potential as the relativistic particles. Note that usually this assumption refers to the nucleons, so the standard formula of the binding energy contains a term that describes the kinetic energy of the nucleons [2-4,19].

A final assumption is that the dissociation of the nucleon mass is spent on the excitation of the kinetic energy of the quarks and the creation of links between the quarks. With all of the assumptions of the binding energy equation of the form

$$
\begin{aligned}
E_b = m_p Z + m_u N - V_u (2Z + N) - V_d (Z + 2N) + Q_u (2Z + N)^2 / r_A + \\
Q_d (Z + 2N)^2 / r_A + Q_{ud} (2Z + N)(Z + 2N) / r_A
\end{aligned}
\qquad (20)
$$

It is indicated: $m_p Z,\ m_n N$ - the total mass of protons and neutrons in nuclei;

V_u, V_d - Chemical potential of u and d quarks, respectively;

$2Z + N$ - the number of u quarks;

$Z + 2N$ - the number of d quarks;

Q_i - Parameters of the interaction of quarks;

r_A - Average radius of the nuclear shells.

To close the model, we set

$$\begin{aligned} & r_A^{\ 3} = A = Z + N, \\ & V_u = \sqrt{m_u^2 + V_0^2 (2Z + N)^{2/3} / (Z + N)^{2/3}}, \\ & V_d = \sqrt{m_d^2 + V_0^2 (Z + 2N)^{2/3} / (Z + N)^{2/3}} \end{aligned} \tag{21}$$

The first equation (21) describes the dependence of the mean radius of the nuclear envelope of the number of nucleons, the second and third of (21) describe the dependence of the chemical potential of a Fermi gas of relativistic particles on their density. The scale factor appearing in the equations (21) let define as the average of the maximum energy of the expressions (18) and (19), thus

$$V_0 = (142.948 + 417.397) / 2 = 280.173 MeV \tag{22}$$

In Figure 1-4 presents the simulation of the binding energy on the equations (20) - (22) for the nuclides of elements with atomic number from 4 to 112, inclusive. Note that the range of the quarks interaction parameters is less than 2 MeV. These parameters depend on the number of protons like the ionization energy of electrons in atoms. It can be concluded that there are quark shells - Figure 5, similar electron shells (note that all the calculations and visualization were carried out on the basis of the system [20]).

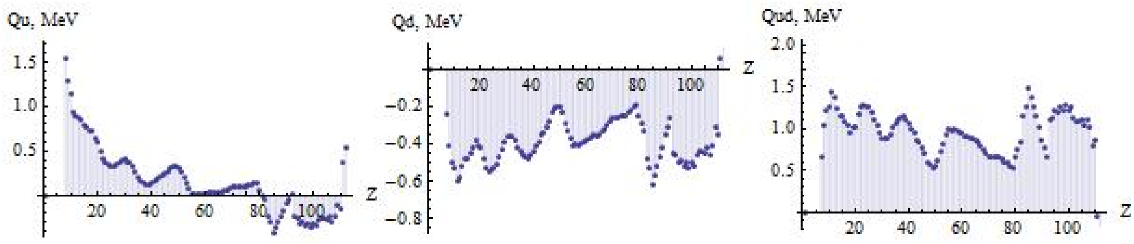

Figure 1: The dependence of the quarks interaction parameters on the number of protons.

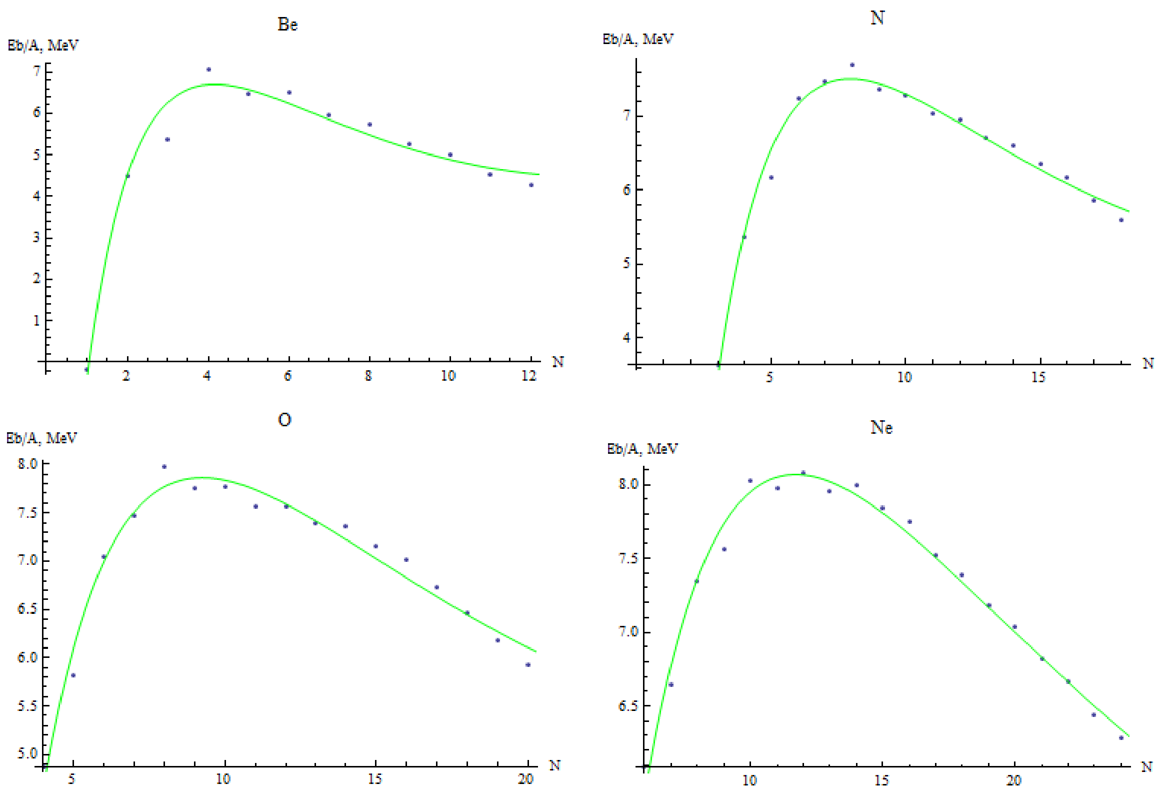

Figure 2: Modeling of the binding energy in the isotopes of beryllium, nitrogen, oxygen and neon – solid lines, and experimental data points.

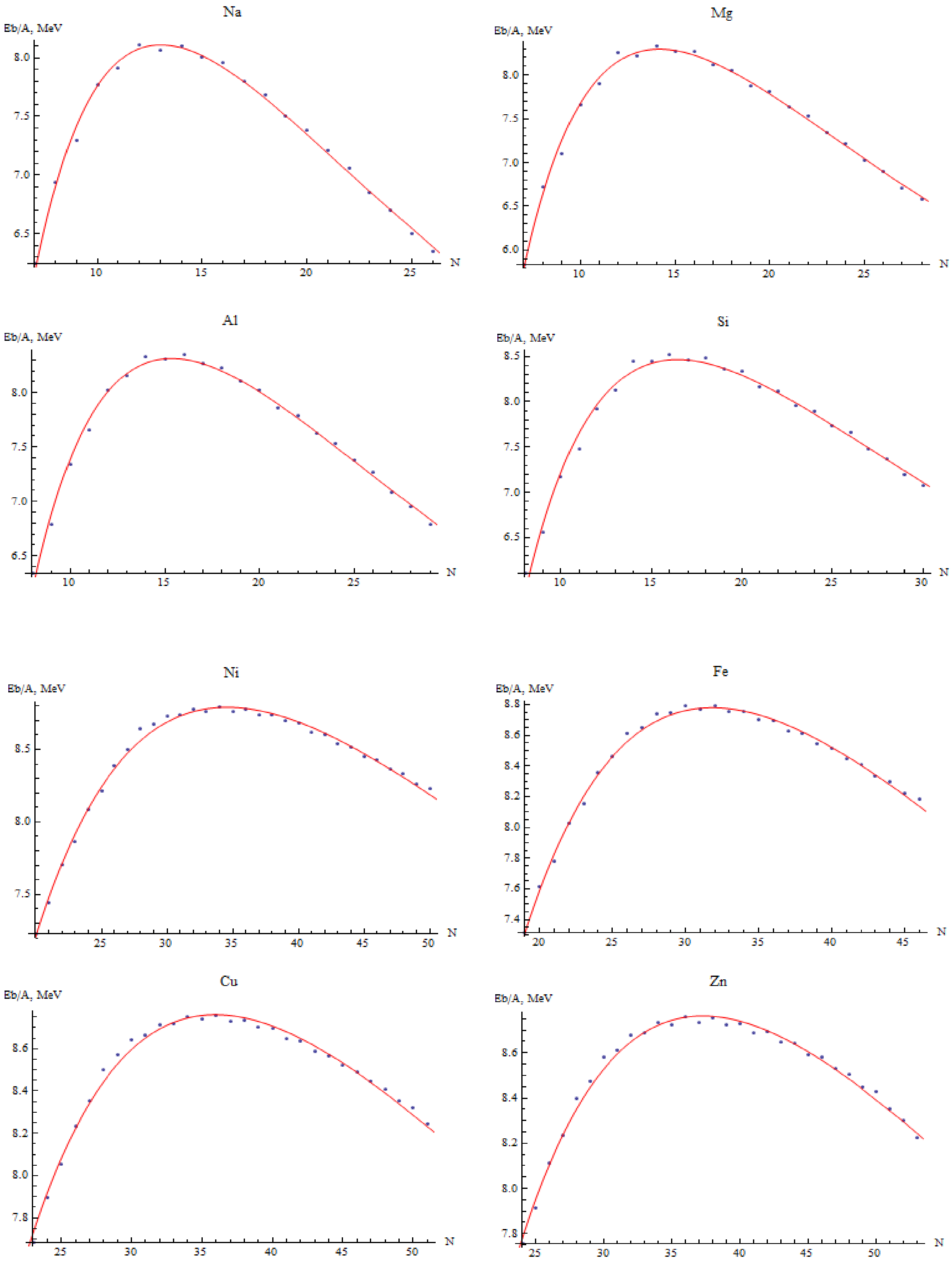

Figure 3: Modeling of the binding energy in the isotopes of sodium, magnesium, aluminum, silicon, nickel, iron, copper and zinc – solid lines, and experimental data points.

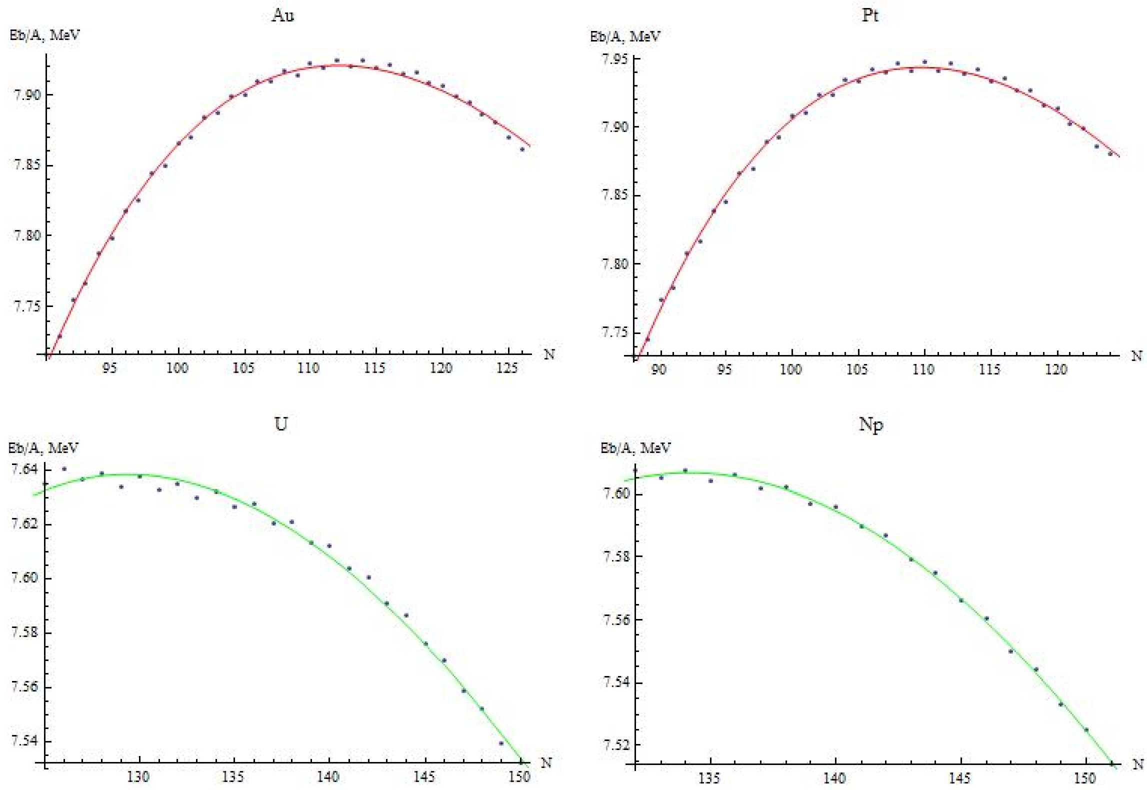

Figure 4: Modeling of the binding energy in the isotopes of gold, platinum, uranium and neptunium – solid lines, and experimental data points.

Quark shell replaces the nuclear shells [1-4], which are consistently filled nucleons. This model assumes that the nucleons dissociate into their constituent quarks, the quarks than fill the shells. This filling process is reflected in the behavior of the interaction parameters – see Figure 1.

Hence the expression of the binding energy (20) has the asymmetry due to the splitting of the masses of nucleons and quarks as well as the asymmetry of the quark interaction in the shells. This asymmetry is mitigated by the large quark chemical potential, which is equivalent to a high kinetic energy of the particles of two Fermi gases, which is, according to (22), about 280 MeV per particle.

Fermi gases consisting of two kinds of quarks, held inside the bubble by gluon wall [7-10, 15-18]. A metric of hadrons has poles inside the wall, which is modeled by Weierstrass function. These poles correspond to an infinite energy, as it follows from (2). Therefore quarks cannot leave the bubble except by crushing the original bubble in two (or more) bubbles. This process corresponds to the decay of nuclei. Along with fragmentation can be considered the process of coagulation of bubbles that corresponding to nuclear fusion.

Clusters of quarks

In the volume of the bubble within the nuclear shell quarks can be combined into clusters that mimic the properties of the proton, neutron and light nuclei - deuterium, tritium, helium, etc. These clusters arise by synchronizing motion of the quarks in the neighboring shells, so, unlike the original particles and nuclei, they are similar to a system of charges that are fixed relative to each other. In Figure 6-8 shows some visualization clusters, made on the basis of equal level surfaces of the electrostatic potential of the charge system using standard software Wolfram Mathematica 9.0 [20]. Potential of a single charge is defined as $\Phi_i = q_i / |\mathbf{r} - \mathbf{r_i}|$, and potential of all charges as $\Phi = \sum_i q_i / |\mathbf{r} - \mathbf{r_i}|$ with charges related to data from Table 2.

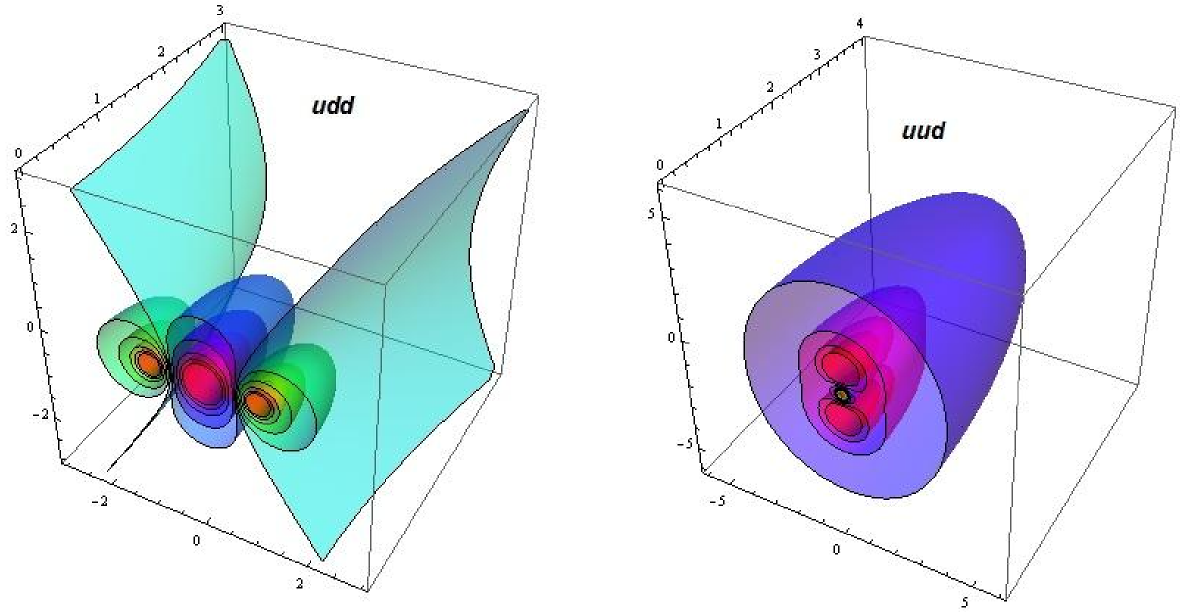

Figure 5: Quark clusters simulating the electrostatic properties of the neutron and the proton; the neutron quark charges {2/3, -1 / 3, -1 / 3} are in the points {{0, 0, 0}, {1, 0, 0}, {-1, 0, 0}}, in the coordinate system {x, y, z}; the proton quark charges {-1 / 3, 2/3, 2/3} are in the points {{0, 0, 0 }, {0, 0, 1}, {0, 0, -1}} in the coordinate system {x, y, z}. Figure shows the levels of the potential {-1, -0.75, -0.5, -0.25, - .1, 0, .1, 0.25, 0.50, 0.75, 1}.

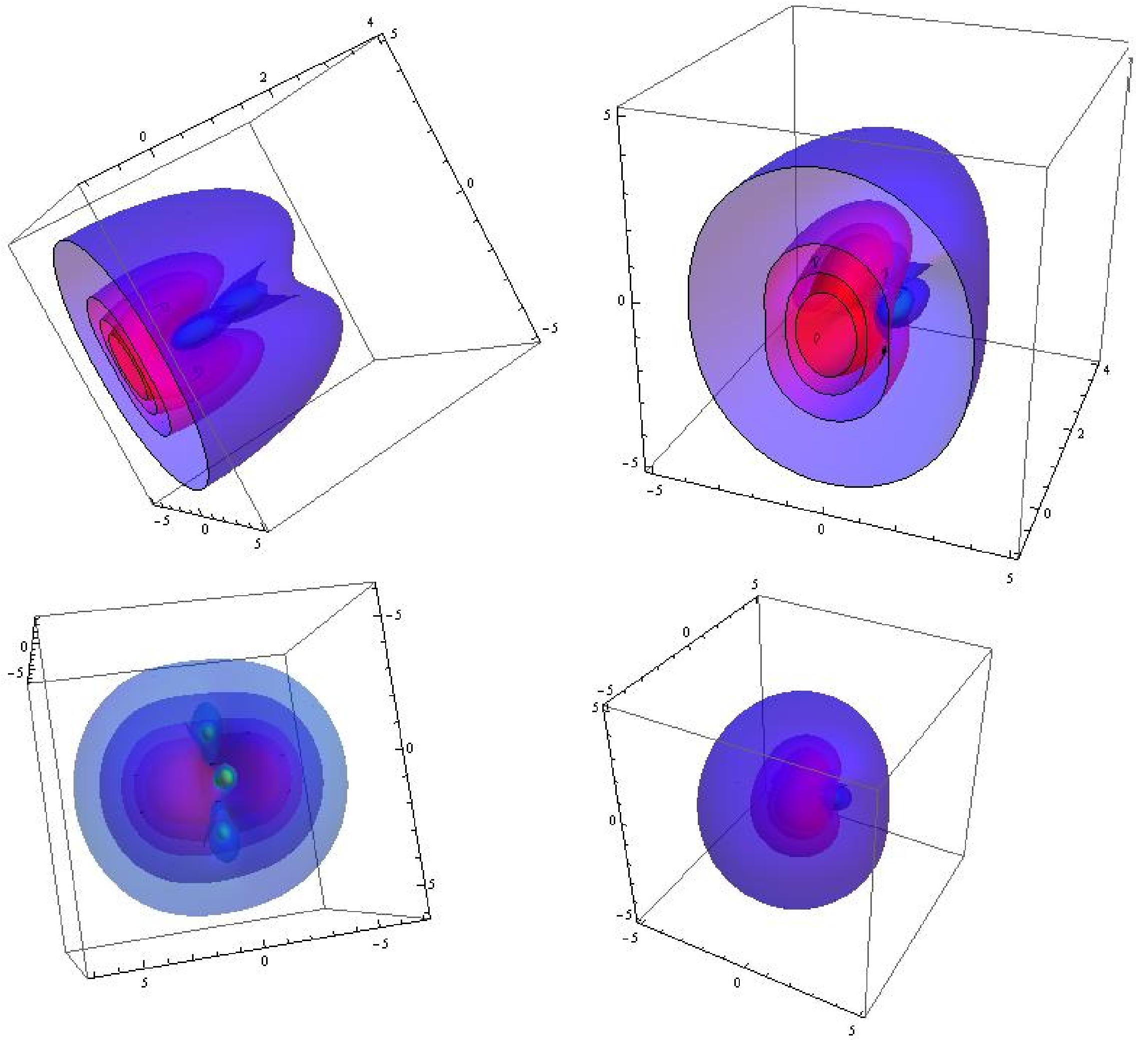

Figure 6: Quark clusters simulating the electrostatic properties of the deuteron; the proton quark charges {-1 / 3, 2/3, 2/3} are in the points {{0, 1, 0}, {0, 0, 1}, {0, 0, 1}}; the neutron quark charges {2/3, -1 / 3, -1 / 3} are located in the points {{0, -1, 0}, {1, 0, 0}, {1, 0, 0}} in the coordinate system {x, y, z}. Surfaces correspond to the levels of the electrostatic potential {-1, -0.75, -0.5, -0.25, -0.1, 0, 0.1, 0.25, 0.50, 0.75, 1}.

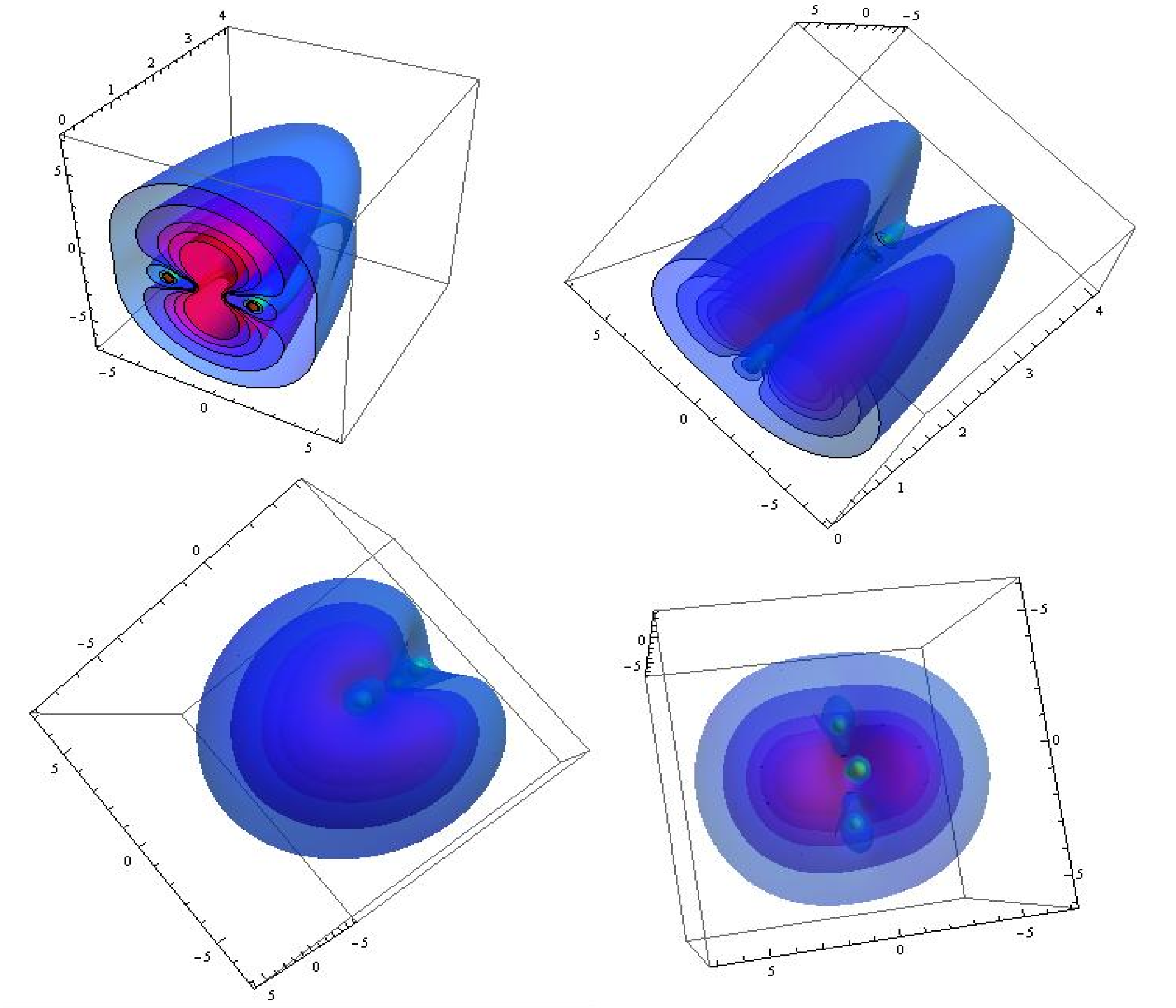

Figure 7: Quark clusters simulating the electrostatic properties of alpha particles; the quark charges of one proton {-1 / 3, 2/3, 2/3} are located at the points {{0, 1, 0}, {0, 0, 1}, {0, 0, -1}}; quark charges of the second proton {-1 / 3, 2/3, 2/3} are located at the points {{0, 2, 0}, {0, 0, 2}, {0, 0, -2}}; the quark charges one neutron {2/3, -1 / 3, -1 / 3} are located at the points {{0, -1, 0}, {1, 0, 0}, {1, 0, 0}}, second neutron quark charges are in the points {{0, 2, 0}, {2, 0, 0}, {2, 0, 0}} in the coordinate system {x, y, z}. Figure shows the levels of total electrostatic potential {-1, -0.75, -0.6, -0.45, -0.35, 0, 0.35, 0.45, 0.60, 0.75, 1}.

The data shown in Figures 6-8 demonstrate that the electrostatic potential of the charge system, consisting of quarks, has a very complex structure, which should be taken into account in the scattering of charged particles. In this regard, we note that the partons or quarks were discovered in experiments on the scattering of electrons on protons [21], as particles constituents of hadrons. Despite the fact that free quarks are not observed, the presence of the complex structure of hadrons can be regarded as an established fact.

Identification of partons with quarks allowed to establish [21] that the u and d quarks have significantly less mass - see Table 2, than anticipated in the original model [12], in which the quark masses greater than the mass of the proton. This choice was due to the modeling of the magnetic moments of baryons [8], but the model presented above shows that the Dirac equation can simulate the magnetic moments of baryons in the form of equation (17), which has the solutions at low mass of the quarks.

On the other hand, the model of the binding energy of the nucleons in nuclei (20) - (22) shows that the energy of quarks in nuclear shells is comparable to the mass of the pion pair. However, in light nuclei of hydrogen isotopes energy quarks seem determined smaller root of eq. (17) [9].

Presence of clusters means that the expansion of the binding energy in powers of the density of the quarks in the right-hand side of equation (20) is present not only linear and quadratic terms, and cubic terms corresponding clusters {uud} and {udd}, but also the terms of the sixth degree, relevant clusters { {uud}, {udd}}, as well as the terms of 12 degrees, corresponding to alpha-particle clusters {{{uud}, {udd}}, {{uud}, {udd}}}.

Indeed, given that the nature of the observed alpha decay, it can be expected that there is a correlation between the terms of 12 degrees with the quark density of the binding energies. This correlation must be especially great for heavy elements. In Fig. 8 shows the curves of the binding energy, calculated from equation (20) without the formation of clusters - the blue curves, as well as the formation of clusters of the deuteron and alpha particles - the red line. The points correspond to the experimental data on the binding energy of the isotopes of lead, mercury, polonium and astatine.

Equation (20) subject to the terms 6 and 12 degrees of density of quarks is of the form

$$E_b / A = m_p n_p + m_n n_N - V_u n_u - V_d n_d + Q_u n_u{}^2 r_A{}^2 + Q_d n_d{}^2 r_A{}^2 \\ + Q_{ud} n_u n_d r_A{}^2 + Q_{np} (n_u n_d)^3 + Q_{npnp} (n_u n_d)^6 \tag{23}$$

Here $n_p = Z / A,\ n_N = N / A, n_u = (2Z + N) / A, n_d = (Z + 2N) / A$ - the density of protons, neutrons, and two types of quarks, respectively; Q_i - the parameters of interaction between quarks, taking into account the contribution of the deuteron cluster and alpha particles in the binding energy.

Note that the expression (23) allows to simulate accurately the energy of the nucleons in nuclei of isotopes of heavy elements. Putting in (23) $Q_u = Q_d = 0$ leads to the model with three parameters - Fig. 9, which highlighted the contribution of the pair interaction of clusters of protons and neutrons. The average value of the interaction of clusters of nucleons is $\langle Q_{np} \rangle = 2.4321 MeV$, which is close to the mass of the u quark. The average value of the interaction of quarks is about $\langle Q_{ud} \rangle = 0.1847 MeV$ only.

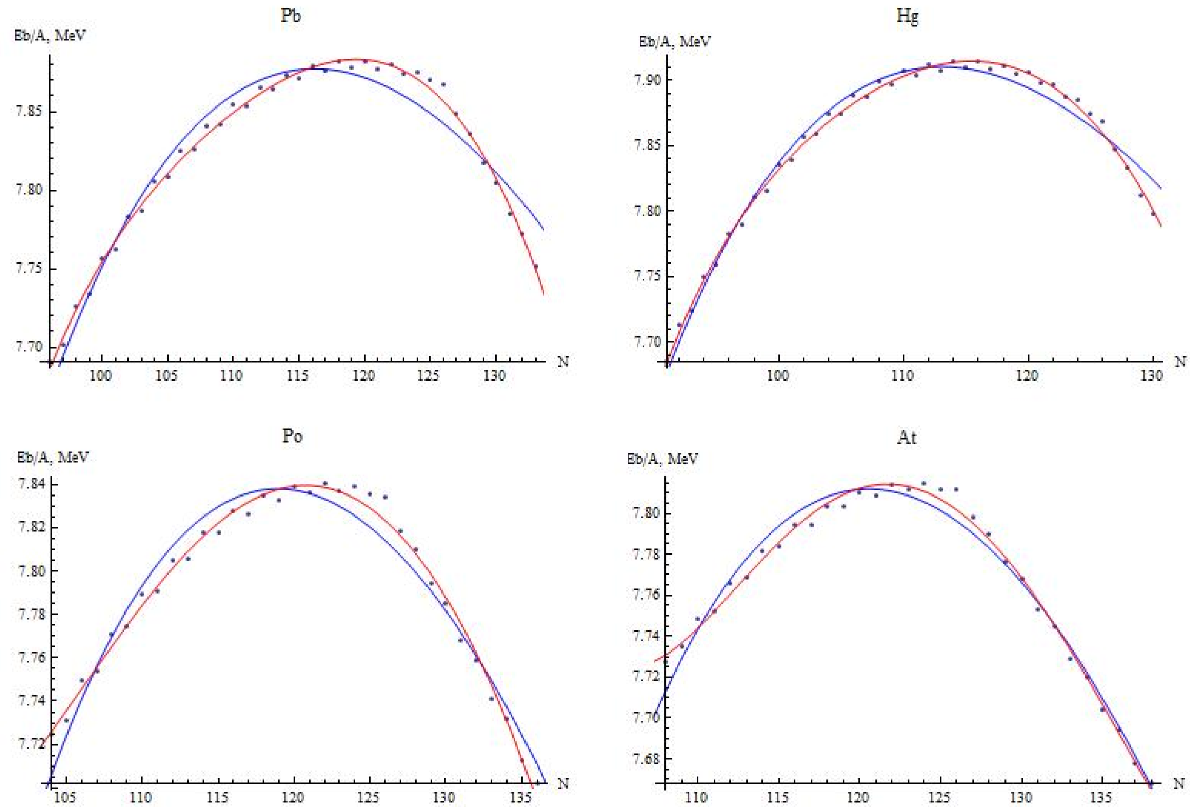

Figure 8: Modeling of the binding energy in the nucleus of lead, mercury, polonium and astatine: blue line is calculated according to equation (20), excluding the clusters; the red line calculated with the cluster deuteron and alpha particles on equation (23). The points correspond to the experimental data.

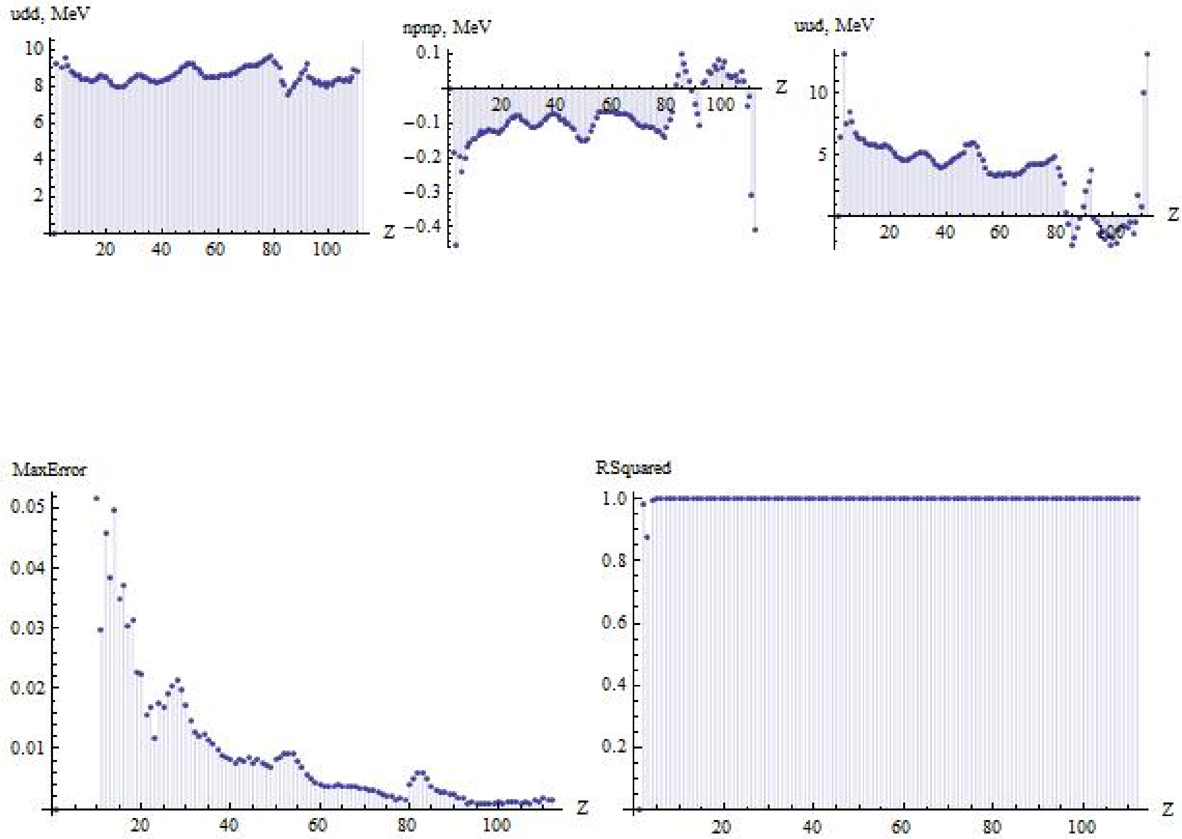

Figure 9: The dependence of the parameters of the interaction of quarks and clusters of nucleons in nuclei of isotopes of different elements on the number of protons.

Structure of quark shells

Build up rule for the quark shells with the cluster formation can be inferred from the above dynamic model. Note that the number of quarks in nuclei is three times greater than the number of nucleons. For example, the nucleus of the isotope of gold contain between 90 and 126 neutrons and 79 protons. Therefore, we must put in the shells from 507 to 615 quarks. Obviously, this task is much greater than the complexity of a similar problem in the placement of 79 electrons in shells of atoms of gold.

This task is complicated for two reasons. First, the quarks in the nucleus have no center of gravity like nuclei in atom, so the radial quantum number is not in the task. Second, the energy of interaction between quarks is small - Fig. 9, while the kinetic energy is high, according to equation (22). Consequently, quark shell formed during the formation of clusters of nucleons, which have high binding energy, but less kinetic energy.

Note that the binding energy of quarks is relatively small, and the binding energy of the nucleon clusters is of the order of quark masses, therefore it can be assumed that the most likely combination is the combination of three quarks. In this form clusters are similar in their properties to the nucleons. According to the first equation (16) the quark density in the inner region of the bubble is constant. Therefore, the state of the system is determined only by the energy. Then the probability of formation of the cluster proportional to the product of the nucleon energy of the three quarks, and the statistics are determined by the Fermi distribution. This leads to the model studied in [22-24]:

$$m_N = m_Z = m_A = q \ln a\,, \quad E_N = E_Z = -E_A / A$$

$$\begin{gathered} x_{i+1} x_i^2 = \frac{K}{e^{-x_i} + a} \\ x_i = -\frac{E_i}{Aq}, \quad K = \frac{4p}{3Aq^3} a g_A (1 + 1/A)^{2/3} b^2(A) b(A+1) \end{gathered} \tag{24}$$

Here $b(A) = 0.05325 \ln A$; g_i, E_i, m_i, q - the weight factor, energy, chemical potential, and temperature of the system accordingly. All dimensional quantities in the model (24) have dimension MeV.

In Fig. 10 shows the bifurcation diagram of the model (24), which defines the rules of quark filling shells. The model (24) predicts that there are only eight quark shells that contain an even number of energy levels. This result can explain the known fact differences nuclei with even and odd number of nucleons. Indeed, for an even number of nucleons is an even number of quarks, so the energy levels in the shells are filled successively by rule 2 +4, and in odd nuclei filled by the rule 1 +2.

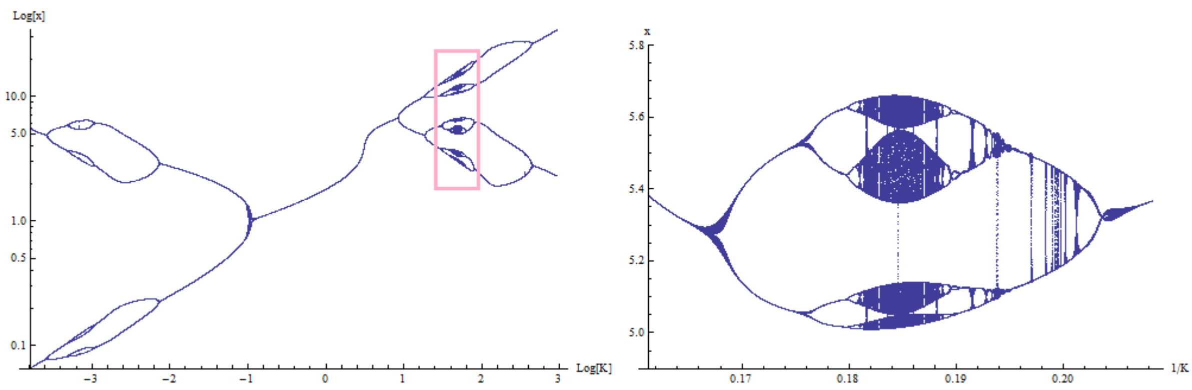

Figure 10: The bifurcation diagram of the model (24) showing the build-up rule of quark shells (highlighted square) and the structure of the third shell in the coordinates (x, 1 / K) for $\alpha = 1/136.2$.

Furthermore, the model (24) may explain why binding energy in the beginning of the neutron addition increases and then decreases - Figure 2-4, 8. This is explained by the form of the quark shell in the plane (E, A) - Fig. 10. It was found that in the model (24) there is a transition to chaotic behavior when the parameter α approaches a certain critical value [22]. Therefore, the model (24) allows us to explain the presence of discrete and continuous spectra of excitation of atomic nuclei. For example, the structure of the third shell energy levels split into two with increasing atomic number (descending order parameter K), and then merge into a continuous spectrum - Figure 10.

Finally, we note that the quark shells model can explain the structure and properties of atomic nuclei, just as the electron shells can explain the structure and properties of atoms.

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