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КВАНТОВАЯ ЭЛЕКТРОДИНАМИКА ЛОРЕНЦА

Трунев Александр Петрович
к.ф.-м.н., Ph.D.
Директор, A&E Trounev IT Consulting, Торонто, Канада

Обсуждается вопрос о расширении электродинамики Лоренца до квантовой теории. Сформулирована система уравнений квантовой электродинамики Лоренца

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LORENTZ QUANTUM ELECTRODYNAMICS

Alexander Trunev
Cand.Phys.-Math.Sci., Ph.D.
Director, A&E Trounev IT Consulting, Toronto, Canada

The question of extending the Lorentz electrodynamics to quantum theory is discussed. The system of equations of the Lorentz quantum electrodynamics was established

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Lorentz electrodynamics (brief overview)

In classical electrodynamics, an electron, just like any other charge, has the electric and magnetic field. To describe self-fields of the electron Lorentz [1] using scalar and vector potentials satisfying the equations:

$$\nabla^2 f - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} f = -r$$
$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A} = -\frac{\mathbf{u}}{c} r \tag{1}$$

Here - $c,\ r,\ u$ the speed of light, the charge density and the velocity of the electron, respectively. In the special case of motion with constant velocity vector potential is expressed through the scalar potential in the form [1]

$$\mathbf{A} = \frac{\mathbf{u}}{c} f \tag{2}$$

Hence, the problem of determining the fields of the electron is reduced to the problem of congestion charges potential with a given density and velocity of center of mass. As it know, to solve this problem, Lorenz [1] used transformation of variables (Lorentz transformation), which allows reducing the problem to the Poisson equation:

$$x' = \frac{x - ut}{\sqrt{1 - b^2}}$$

$$r' = r\sqrt{1 - b^2}, \quad f' = f\sqrt{1 - b^2} \tag{3}$$

Here $\beta = u / c$.

$$\frac{\partial^2 f'}{\partial x'^2} + \frac{\partial^2 f'}{\partial y^2} + \frac{\partial^2 f'}{\partial z^2} = -r' \tag{4}$$

Thus, the problem of finding the potential of moving charges reduced to the problem of finding the electrostatic potential of fixed charges with a given density. Indeed, the unknown electric and magnetic fields are determined by potential gradients, which is the solution of equation (4), we have

$$\frac{\partial f}{\partial x} = (1 - b^2)^{-1}\frac{\partial f'}{\partial x'}, \quad \frac{\partial f}{\partial y} = (1 - b^2)^{-1/2}\frac{\partial f'}{\partial y},$$

$$\frac{\partial f}{\partial z} = (1 - b^2)^{-1/2}\frac{\partial f'}{\partial z}$$

$$A_x = bf, \quad A_y = A_z = 0$$

$$\frac{\partial A_x}{\partial t} = -b^2 c\frac{\partial f}{\partial x}$$

Hence we find the electric and magnetic fields by formulas

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla f = -\begin{pmatrix} (1-b^2)f_x \\ f_y \\ f_z \end{pmatrix}, \quad \mathbf{H} = \nabla \times \mathbf{A} = b\begin{pmatrix} 0 \\ -f_z \\ f_y \end{pmatrix}$$

The transition from the system (1) to (4) actually is a Lorentz transformation. This transformation includes the Galileo transformation, $x \to x - ut$, as well as the similarity transformation (3), familiar from relativistic theories.

However, the charge density remains an unknown quantity, which in Abraham theory [2] is replaced by the surface charge distribution on a rigid sphere, i.e. boundary condition. This approach allows us to solve completely the problem of the electric and magnetic field of an electron moving with constant velocity [1-2]. Indeed, the initial rigid sphere under the transformations (3) is transformed into an ellipsoid for which the solution of electrostatic problem is well known. Using this solution, Abraham [2] calculated the energy and momentum of the electromagnetic field of an electron in the form of

$$E = \frac{e^2}{8pRb}\left(\ln\frac{1+b}{1-b} - b\right)$$

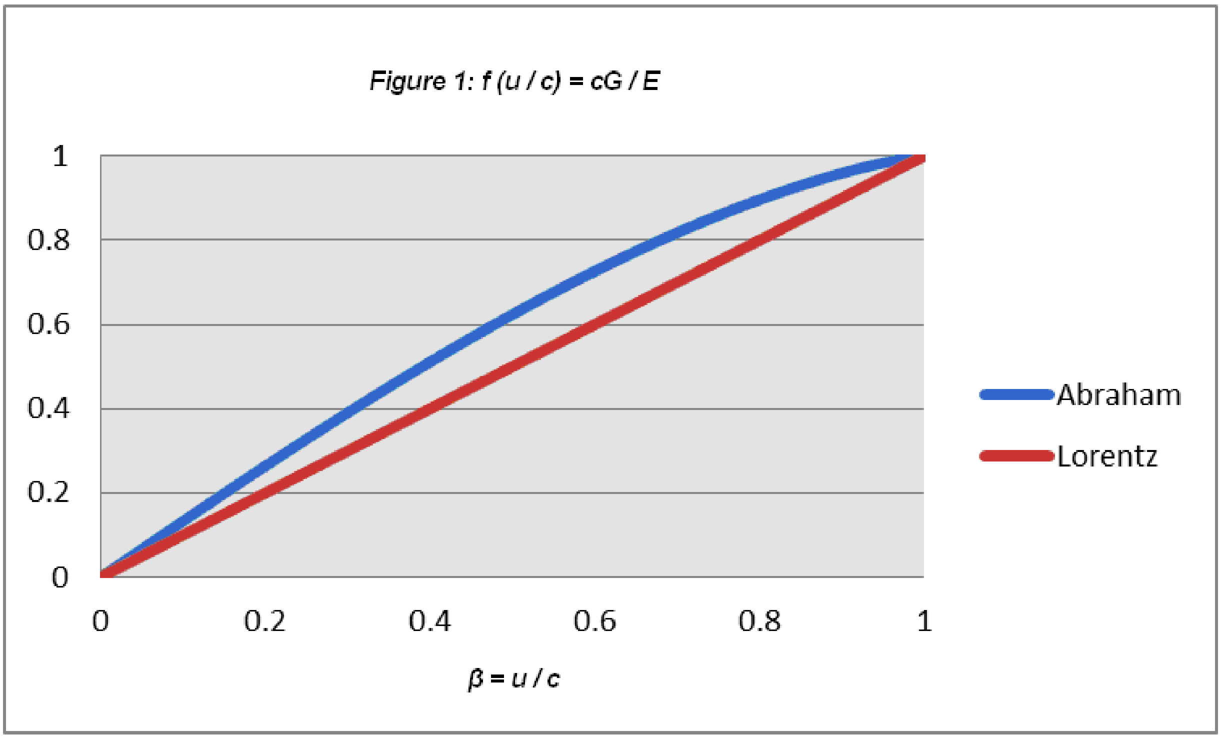
$$G = p_x = \frac{e^2}{8pRcb^2}\left(\frac{(1+b^2)}{2}\ln\frac{1+b}{1-b} - b\right) \tag{5}$$

Here R - radius of the sphere.

The limiting value of the total energy of the electron is at $b \to 0$

$$E_0 = E(0) = \frac{e^2}{8pR}$$

Combining the first and second equation (5) we find the dispersion relation, which we write in the form $f\,(u\,/\,c) = cG\,/\,E$ – see Figure 1.



Using the momentum expression (6), we can determine the longitudinal and transverse mass, according to the formulas

$$m_l = \frac{1}{c}\frac{dG}{db} = \frac{e^2}{6pRc^2 b^3}\left(\frac{2b}{1-b^2} - \ln\frac{1+b}{1-b}\right)$$

$$m_t = \frac{1}{c}\frac{G}{b} = \frac{e^2}{6pRc^2 b^3}\left(-2b + (1+b^2)\ln\frac{1+b}{1-b}\right) \qquad (6)$$

The longitudinal mass characterizes the inertia of the body when the velocity changes in magnitude, and the transverse mass characterizes the inertia only when the velocity direction changes, for example, an electron moving in a uniform magnetic field. Limit values for longitudinal and transverse mass (6) coincide at $b \to 0$, in this case we have

$$m_l = m_t = \frac{e^2}{6pRc^2} \tag{7}$$

Assuming that the mass limit (7) is equal to the rest mass, we can define the classical electron radius

$$R_e = \frac{e^2}{6pm_0c^2} \tag{8}$$

Kaufman [3] performed a number of experiments in which he measured deflection of beta-electrons in the electric and magnetic field. He showed that the experimental dependence of the transverse mass on the velocity corresponds well to the Abraham theory – see Figure 2. Kaufman believed that the electron mass is entirely of electromagnetic energy, as follows from expressions (5) - (6).

Lorentz theory of electrons [1], as well as Abraham theory [2], based on the idea of the existence of a luminiferous ether - a continuous medium in which electromagnetic waves propagate. The negative result of Michelson-Morley experiment to detect the influence of the earth's motion through the ether on the speed of light, forced to reconsider the basis of Lorentz theory of electrons [1[. As a result, Lorentz formulated a general form of the electrodynamics equations transformation, which, together with the transformations (3) include the conversion time by the formula

$$t' = \frac{t - bx/c}{\sqrt{1-b^2}} \tag{9}$$

Lorenz suggested that all material bodies are experiencing reduction in the size along the direction of movement by the formula (3), which in this case can be written as

$$l = l'\sqrt{1-b^2} \tag{10}$$

The new model is the electron conducting sphere in the coordinate system, where it rests, and in the moving frame the electron is an ellipsoid of rotation, compressed into the direction of motion according to the equation (10). Lorenz found that, in this case, the longitudinal and transverse electron mass is converted asas follows

$$m_l = m_0(1-b^2)^{-3/2}, \quad m_t = m_0(1-b^2)^{-1/2} \tag{11}$$

Lorenz has shown that these equations are applicable not only to electrons and atoms, but also to any material bodies.

Einstein [4-5] developed Lorenz ideas, basing them on the principle of relativity and the constancy of the speed of light. Once published the first paper on relativity theory [4], Kaufmann repeated his experiments to compare the theory by Abraham [2], Bucherer [6] and Lorentz-Einstein [1, 4]. Although the results of Kaufman [3] more consistent with the Abraham theory [2], nevertheless Lorenz [1] and Einstein [5] praised these experiments, never doubting their authenticity. In subsequent years, was put a lot of experiments [7-9] that confirmed the theory of Lorentz-Einstein. However, the Abraham theory was rejected as a bankrupt after the publication of data [9], obtained in the electrostatic analyzer, which contained only three points - see Figure 2.

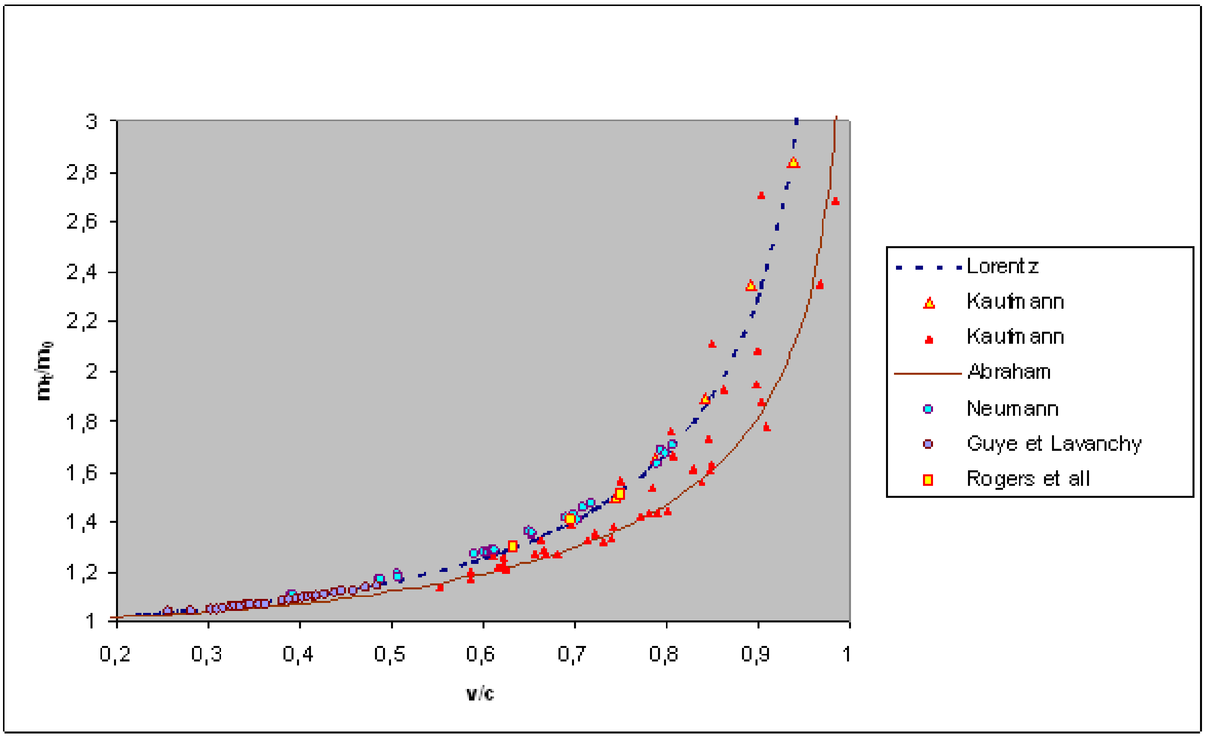


Figure 2: Dependence of the transverse electron mass on velocity according to the Abraham theory [2] and Lorentz-Einstein theory [1, 4], and according to experimental data [3, 7-9].

At the time, it suggested, rather, a loss of interest in the problem, rather than the desire to reconcile the Lorentz-Einstein theory with experiment. Indeed, at the time of publication [9] in 1940 already existed Dirac relativistic quantum theory of the electron [10], and the spectrum of the hydrogen atom was explained on the basis of the nonrelativistic Schrödinger equation [11]. The special theory of relativity (STR) was accepted by leading theoretical physicists without further discussion, as the basis for the construction of elementary particle physics.

The basis of Dirac's relativistic quantum theory [10] is STR, derived by Einstein [4], and based on the analysis of Maxwell-Hertz equations and Lorentz electrodynamics [1], in which the electron is described by equations (1). But the data in Figure 2, painstakingly collected by a whole generation of experimentalists to confirm Lorentz-Einstein theory, obtained by analyzing the trajectories of electrons, rather than spinors in the Dirac theory [10]. If, however, perform data analysis [3, 7-9] on the basis of the Dirac theory, we obtain the unexpected result that the scatter of the data in Fig. 2 may be associated with the excitation frequency of natural oscillations of electrons in a magnetic field [12] - Figure 3.

In this sense, data [3, 7-9] and others can be regarded as fit to a known result, by adjusting the setting on the playback of that particular frequency of quantum oscillations. Consequently, the Dirac theory is at odds with the original hypothesis of Lorentz, so it does not allow testing this hypothesis experimentally. For example, it is impossible to distinguish the Abraham theory [2] from the Lorentz theory [1], which is equally consistent with the theory of Dirac, which, in turn, describes well the entire set of known experimental data - see Fig. 3.

Another apparent paradox is that, apart quantum mechanics and electrodynamics are the linear theory, which holds the principle of superposition, whereas the quantum electrodynamics (QED), which unites the Dirac theory and the electrodynamics, is a nonlinear theory / 13 /.

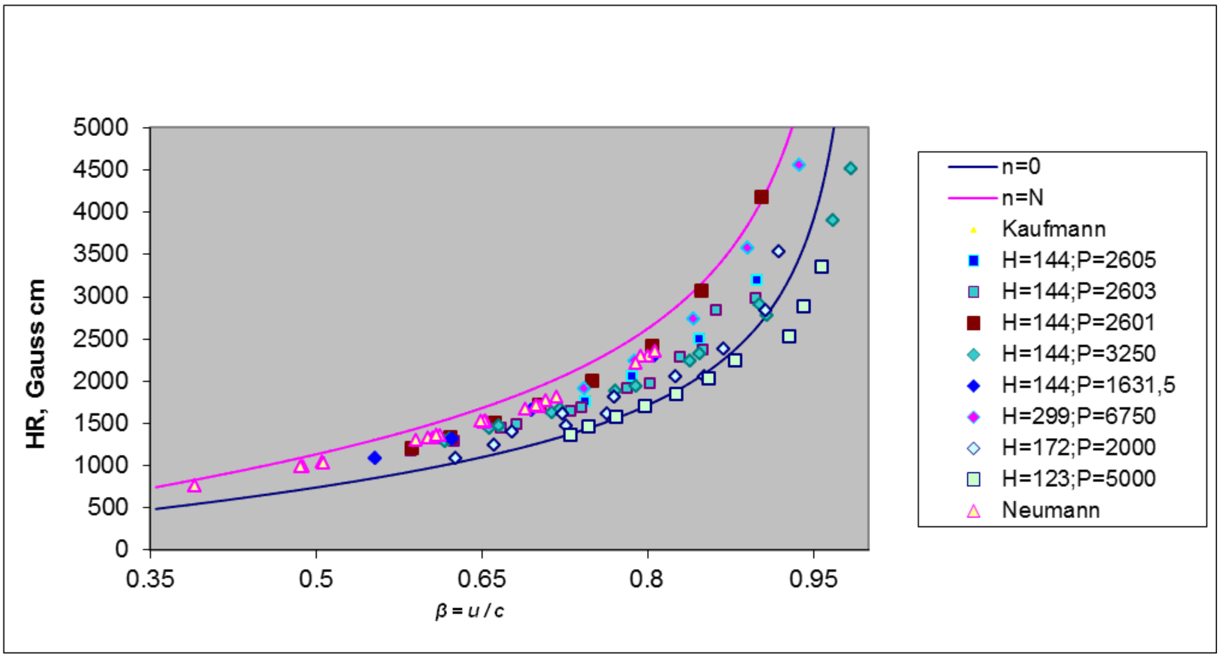


Figure 3: Dependence of the electron momentum on the velocity in a homogeneous magnetic field calculated for two quantum numbers [12], and according to experimental data [3, 7].

Thus, historically Lorentz electrodynamics provided the basis for the withdrawal of the Lorentz transformation, based on which, in turn, there was the Dirac relativistic quantum theory [10]. Consequently, these theories must be linked, as describe one particle - an electron.

Mills theory and its generalization

For paradoxes described above was found an unexpected solution in the Mills theory [14]. Mills main hypothesis [14] is that the electric charge density in the right-hand side of equations (1) is described by the wave equation of the form

$$\nabla^2 r - \frac{1}{v^2}\frac{\partial^2}{\partial t^2} r = 0 \qquad (12)$$

Here – *v* speed charge-density waves.

The hypothesis (12) together with the assumption of the charge distribution in the atom on the surface of the sphere of fixed radius, determined from Bohr’s theory, allows one to calculate the energy levels of multielectron atoms and to determine the anomalous magnetic moment of the electron [14]. In this sense, the Mills theory is an alternative to quantum electrodynamics (QED), as it allows the calculation of all relativistic quantum effects with the same accuracy as QED, but it is a linear theory.

The basic equation (12) of Mills theory, which he used to solve the problems of atomic physics, chemistry, and physics of elementary particles, at first, not directly related to quantum mechanics. Moreover, Mills asserts that quantum mechanics is a profoundly erroneous theory that contains internal logical contradictions. In reality, however, it is easy to show using the results of our paper [12] that the equation (2) can be obtained from the relativistic quantum Dirac equation – see [15]. Indeed, as shown in [12], the charge density and potential of an electron subject to both of the Dirac equation and the Lorenz equations (1) are related by:

$$r = \left(-\frac{e}{\mathbf{h}^2 c^2}\left(\tilde{f}E - c(\tilde{\mathbf{A}}\mathbf{p})\right) + \frac{e^2}{\mathbf{h}^2 c^2}\left(\tilde{f}^2 - \tilde{\mathbf{A}}^2\right) - \frac{m_0^2 c^2}{\mathbf{h}^2} \right) f \qquad (13)$$

It is indicated $E, \mathbf{p}$ is the energy and momentum of electron, $\tilde{f}, \tilde{\mathbf{A}}$ - the scalar and vector potential of the external electromagnetic field. Hence we find that in the absence of external fields, the charge density depends linearly on the potential of the electron:

$$r = -\frac{m_0^2 c^2}{\mathbf{h}^2} f \qquad (14)$$

Substituting this expression into the first equation (1), we finally obtain

$$\nabla^2 r - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} r = \frac{m_0^2 c^2}{\mathbf{h}^2} r \qquad (15)$$

Mills used his basic equation (12) only to find the singular solutions in which the density distribution along the radial coordinate is described by the Dirac delta function. On such solutions eq. (15) takes the Mills form (12)

$$\nabla^2 r - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} r = 0 \qquad (16)$$

We give a simple derivation of equation (13), based on a special recording of the Dirac equation in the form of second-order equation / 13 /

$$\left(\left(\frac{i\hbar}{c}\frac{\partial}{\partial t}-\frac{e}{c}\tilde{f}\right)^2-\left(i\hbar\nabla+\frac{e}{c}\tilde{\mathbf{A}}\right)^2-m_0^2c^2+\frac{e\hbar}{c}\hat{\Sigma}\tilde{\mathbf{H}}-i\frac{e\hbar}{c}\hat{\boldsymbol{\alpha}}\tilde{\mathbf{E}}\right)\psi=0 \qquad (17)$$

Here $\tilde{\mathbf{E}}, \tilde{\mathbf{H}}$ is the electric and magnetic field of an external source, and the corresponding terms in equation (17) describe the interaction of the electron spin with an external electromagnetic field. Typically, the energy of this interaction is relatively small, so it can be neglected. As a result, equation (17) reduces to the Klein-Gordon

$$\left(\left(\frac{i\hbar}{c}\frac{\partial}{\partial t}-\frac{e}{c}\tilde{f}\right)^2-\left(i\hbar\nabla+\frac{e}{c}\tilde{\mathbf{A}}\right)^2-m_0^2c^2\right)\psi=0 \qquad (18)$$

The resulting equation contains only the identity matrix, so it reduces to a system of four independent equations. But the system (1) also consists of four independent equations, so between the solutions of (1) and (18) must be a relationship, as they describe the state of the same physical object - an electron. Assuming that this relationship is linear, and that Lorenz calibration is performed, we arrive at equation (13). Note that in quantum electrodynamics, current and wave function are interconnected nonlinear way. It is not known, however, whether this connection is always to be nonlinear. Mills theory [14], based on the linear equation (12), allows us to predict the anomalous magnetic moment of the electron and the muon, and the Lamb shift with the same accuracy as QED, but without mathematical artifices peculiar to this theory.

Lorentz quantum electrodynamics

We formulate a complete system of equations of the Lorentz linear quantum electrodynamics,

$$\nabla^2 f - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} f = -r$$

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A} = -\frac{\mathbf{u}}{c} r \tag{19}$$

$$r = \left(-\frac{e}{\mathbf{h}^2 c^2}\left(\tilde{f}E - c(\tilde{\mathbf{A}}\mathbf{p})\right) + \frac{e^2}{\mathbf{h}^2 c^2}\left(\tilde{f}^{\,2} - \tilde{\mathbf{A}}^2\right) - \frac{m_0^2 c^2}{\mathbf{h}^2} \right) f$$

According to the rule of the operators of momentum and energy we put in the right-hand side of equation (19)

$$E \to i\mathbf{h}\frac{\partial}{\partial t}, \quad \mathbf{p} \to -i\mathbf{h}\nabla \tag{20}$$

Undefined value is the electron velocity on the right side in the second equation (19). We use the fact that the Lorentz transformation leads to expression

$$\mathbf{u} = \frac{\mathbf{p}}{E}c^2$$

Multiply both sides of the second equation (19) to E on the left, as a result we obtain

$$E\nabla^2 \mathbf{A} - \frac{E}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A} = -E\frac{\mathbf{u}}{c} r = -c\mathbf{p} r$$

Replacing in this equation, the energy and momentum to the operators of differentiation according to (20), we find

$$\nabla^2 \mathbf{A}_t - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A}_t = c\nabla r \tag{21}$$

We use the expression of the electric field through electromagnetic potentials, we have

$$\mathbf{E} = -\nabla f - \frac{1}{c}\mathbf{A}_t$$

We express here the derivative of the vector potential with respect to time and substitute into equation (21). As a result, considering the first equation (19), we finally find that the electric field of the electron is described by the wave equation

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = 0 \tag{22}$$

Equation (22) must be supplemented by a standard equation for the divergence field, which, taking into account the expression of charge density has the form

$$\nabla \cdot \mathbf{E} = \left(-\frac{e}{\mathbf{h}^2 c^2}\left(\tilde{f}E - c(\tilde{\mathbf{A}}\mathbf{p})\right) + \frac{e^2}{\mathbf{h}^2 c^2}\left(\tilde{f}^{\,2} - \tilde{\mathbf{A}}^2\right) - \frac{m_0^2 c^2}{\mathbf{h}^2} \right) f$$

Note that the transition from the second equation (19) to (22) fell from the description of the magnetic field of the electron. To restore the symmetry inherent in Maxwell equations, apply the operation of the rotor to the third equation (19), as a result we obtain

$$\nabla^2\mathbf{H} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{H} = -\frac{1}{c}\mathbf{u} \times \nabla r \tag{23}$$

It is indicated $\mathbf{H} = \nabla \times \mathbf{A}$.

Next, we use the relation between the momentum, velocity and energy in the Lorentz-Einstein theory. Multiplying the left and right side of equation (23) on *E* to the left, as a result, we have

$$E\nabla^2\mathbf{H} - E\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{H} = -\frac{1}{c}E\mathbf{u}\times\nabla r = -c\mathbf{p}\times\nabla r$$

Finally, replacing the operators of momentum and energy due to equations (20), and noting that the curl of the gradient of any function is zero, we finally find

$$\nabla^2\mathbf{H}_t - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{H}_t = 0 \tag{24}$$

Given the fact that in quantum mechanics an electron cannot be represented as a static source, it is possible to reduce the order of (24), so along with equation (24) holds and the standard equation describing the magnetic field of electromagnetic waves

$$\nabla^2\mathbf{H} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{H} = 0 \tag{25}$$

Finally, used the definition of the vector magnetic field, we find the last equation in the Lorentz quantum electrodynamics,

$$\nabla\cdot\mathbf{H} = 0 \tag{26}$$

Next, we note that the first equation (19) with regard to the expression of the charge density given by the third equation (19) reduces to the Klein-Gordon equation (18). Therefore, the complete system of equations of the Lorentz quantum electrodynamics has the form

$$\left(\left(\frac{i\mathbf{h}}{c}\frac{\partial}{\partial t} - \frac{e}{c}\tilde{f}\right)^2 - \left(i\mathbf{h}\nabla + \frac{e}{c}\tilde{\mathbf{A}}\right)^2 - m_0^2c^2\right)f = 0 \tag{27}$$

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = 0$$

$$\nabla^2 \mathbf{H} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{H} = 0$$

$$\nabla \cdot \mathbf{E} = \left(-\frac{e}{\mathbf{h}^2 c^2}\left(\tilde{f} E - c(\tilde{\mathbf{A}}\mathbf{p})\right) + \frac{e^2}{\mathbf{h}^2 c^2}\left(\tilde{f}^2 - \tilde{\mathbf{A}}^2\right) - \frac{m_0^2 c^2}{\mathbf{h}^2} \right) f$$

$$\nabla \cdot \mathbf{H} = 0$$

Here the first equation describes the electron interaction with the external field, the second and third equations describe the electromagnetic radiation, and the fourth equation describes the distribution of the electron charge, due to quantum effects. As follows from equations (27) to describe the electron in Lorentz quantum electrodynamics, does not require a wave function, as the system of equations (23) is closed, and the parameters of the theory have a clear physical meaning.

Numerous applications of the Lorentz quantum electrodynamics to the description of various physical phenomena can be found in the monograph [14]. It may be noted that the agreement between Mills theory and experiment is sufficiently accurate, as in standard QED, but due to the linearity of the theory is achieved by the extraordinary simplicity of the description of the basic quantum relativistic effects.

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