УДК 372.8

13.00.00 Педагогические науки

ОБ ЭКСПЕРИМЕНТАЛЬНЫХ ВОЗМОЖНОСТЯХ СИСТЕМЫ ДИНАМИЧЕСКОЙ ГЕОМЕТРИИ GEOGEBRA

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В статье представлены возможности системы динамической геометрии GeoGebra, которые могут быть использованы для проведения экспериментальной исследовательской работы. Компьютерный эксперимент как метод исследования служит для открытия новых знаний, проверки гипотез, вовлечения обучаемых в активный процесс познания нового материала. В качестве компьютерной технологии реализации компьютерного эксперимента выбрана свободно-распространяемая система динамической геометрии GeoGebra, имеющая русскую версию и дружелюбный интерфейс, которую удобно применять как на уроках, так и дома. Описаны возможности инструментов, используемых для реализации компьютерного эксперимента, таких как инструментов для выявления метрических и позиционных свойств объекта, получения сведений об отношении метрических и позиционных свойств объектов, параметрического задания изменений величины, создание таблиц экспериментальных данных. Экспериментальные возможности системы динамической геометрии GeoGebra раскрыты на примерах решения некоторых задач элементарной геометрии о самом простом из многоугольников – треугольнике, который привлекал внимание как ученых древности (Менелай и др.), так и ученых более близких к нашему времени (Эйлер, Понселе и др.). На примере теоремы Менелая, представленной в более доступной форме, чем в учебнике геометрии 10-11 кл. (авторы Л.С. Атанасян, В,Ф,Бутузов и др., продемонстрированы возможности системы по созданию динамических чертежей, таблиц экспериментальных данных и их применения в проведении исследовательской работы

Ключевые слова: СИСТЕМЫ ДИНАМИЧЕСКОЙ ГЕОМЕТРИИ, GEOGEBRA, КОМПЬЮТЕРНЫЙ ЭКСПЕРИМЕНТ, ИССЛЕДОВАТЕЛЬСКАЯ ДЕЯТЕЛЬНОСТЬ, ТЕОРЕМА МЕНЕЛАЯ

UDC 372.8

Pedagogical sciences

ABOUT EXPERIMENTAL OPPORTUNITIES OF DYNAMIC GEOMETRY SYSTEM CALLED GEOGEBRA

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The article presents the possibilities of the GeoGebra dynamic geometry environment which can be used for conducting experimental research work. Computer experiment as a method of research serves to discover new knowledge, to test hypotheses, to involve students in the active process of learning new material. Freely-distributed GeoGebra's dynamic geometry environment which has a Russian version and a friendly interface that can be conveniently used both in classrooms and at home, is chosen as a computer technology for implementing a computer experiment. There are described possibilities of the tools used to implement the computer experiment. Among them are tools for detecting the metric and positional features of the object, obtaining information about the relationship between metric and positional features of objects, a parametrical task of changes of size, creation of experimental data tables. The experimental capabilities of the GeoGebra dynamic geometry environment are revealed by the examples of solving some elementary geometry problems about the simplest of polygons - a triangle that attracted the attention of both ancient scientists (Menelaus and others) and scientists who are closer to our time (Euler, Poncelet et al.). Capabilities of the system for creating dynamic drawings, tables of experimental data and their application in conducting research work are demonstrated on the example of Menelaus' theorem presented in a more accessible form than in the textbook of geometry 10^{th} -11^{th} forms (authors L.S. Atanasyan, V. F. Butuzov, etc.)

Keywords: DYNAMIC GEOMETRY ENVIRONMENT, GEOGEBRA, COMPUTER EXPERIMENT, RESEARCH ACTIVITY, MENELAUS THEOREM

Doi: 10.21515/1990-4665-128-018

1. Introduction

One of the requirements of the Federal State Educational Standard of the general education of the new generation is the active use ideas of design and research training in secondary school which can be implemented at the expense of extracurricular activities of pupils in the preparation and implementation of educational and research projects as well as through the incorporation of research training elements in subject preparation [1].

In a pattern basic educational program of basic education Mathematics is regarded not only as the main subject in mastering the basics of theoretical research but also as an area for formation of skills to conduct experiments and research in virtual laboratories, acquisition of skills, processing and analysis of empirical data knowledge-based statistical elements. Thus, the new requirements of the of the Federal State Educational Standard set before the general mathematical education system a problem of formation by the students some research experience both theoretical mathematician and mathematician-experimenter.

The idea of a research approach in training mathematics is not new. One of the first M.V. Lomonosov realized this idea in practice. In the middle of the XVIII century he introduced the «experimental method» in teaching Physics and Mathematics for gymnasium pupils at the Academy of Sciences of St. Petersburg and then at the University of Moscow justifying the need for its use with the fact that the whole process of human knowledge is determined by the needs of practical activity.

M.V. Lomonosov proposed to begin explanation of the new material with setting special demonstration experiments making the truth of scientific statements visual that would encourage the development of interest and demand on knowledge [2, c. 23].

The idea of research training in mathematics was born in the XVIII century as an idea of rapprochement with elements of training of scientific research. Then it was developed in the XIX and XX centuries by improving the forms, methods and means of its implementation in educational practice [3, c. 40].

In 1960 our national well-known pedagogue B.E. Raikov proposed a new term «research method» as «... a method of reasoning from particular facts, independently observed and studied by schoolchildren» [4, c. 328]. He identified the following stages in the application of the research method:

1) observation and statement of questions;
2) construction of assumed decisions;
3) research of assumed decisions and choice one of the most verisimilar;
4) verification of the hypothesis and its final statement.

Recent years new research training models of teaching mathematics began to appear due to the implementation of international scientific and educational projects. Among these projects we can draw attention to the project «Fibonacci» [5], the project «Developing Key Competences by Mathematics Education» [6], the project «Mathematics and Science for Life» [7], the Russian-Bulgarian project «Methods and Information Technologies in Education» [8].

To implement the idea of a research approach in teaching mathematics one start to use computer technology widely. Earlier experiments were time-consuming to implement them in practice of mathematics teaching but with the advent of computer technology the feasibility of computing and graphics experiments has become not only possible but also exciting for pupils.

The appearance of automated scientific research systems in the late 70s of the 20th century led to the spread of an experimental approach in mathematics that brought the mathematics methodology closer to the methodology of the natural sciences. The concept of «computer experiment» appeared.

Computer experiment was originally understood as a model experiment in which the object of research is completely modeled in digital form on a computer. The term «computer experiment» came into the methodological science in connection with the solution of the problem of informational support of education. Most researchers consider it as a kind of model experiment where a virtual model acts as a substitute for a real object or process. There is also a point of view for a computer experiment as «..an experiment with data on the results of monitoring the behavior of the system being studied, stored in electronic work sheets in order to predict the behavior of given and similar systems outside the observation area» [9, с. 68].

Computer experiments have many advantages over the natural experiment:

– low cost of computer experiment (computer / computers are needed);

– possibility of multiple repetition of the research task for various initial data;

– possibility of obtaining a large amount of data about the behavior and features of the object for a short period of time;

– possibility of performing the experiment both in the academic and extra-curricular time.

Relating to the method of teaching mathematics, we can consider that a computer experiment is the manipulation with virtual models of mathematical objects regardless of their complexity and scale, accompanied by either the data gathering of the objects features being studied with their setting or by observing the nature of the change of these features on the computer screen with subsequent analysis of the received results. In view of the fact that computer mathematical experimentation supposes the practice of obtaining knowledge about the object of research, identifying features and dependencies in this object, computer experiment can be a means of obtaining and mastering knowledge in mathematics [10, с. 458].

Thus, a computer experiment allows (Fig. 1):

– to determine the characteristics of the object in accordance with given conditions;

– to reveal the features and dependencies in the object under certain additional conditions;

– to confirm or refute the hypothesis of the research [11, c. 23].

In mathematical science the relation to computer experiments is ambiguous. Despite the possibility of obtaining a visual confirmation of the truth of the scientific factors, there remain doubts about the reliability of the obtained results and the importance of their logical proof.

As I.F. Sharygin marked: «... a computer is a very useful tool in geometric researches. With its help it is possible to discover new interesting geometric facts experimentally. The most important role to prove these facts (only!) remains for a person» [12, c. 51].

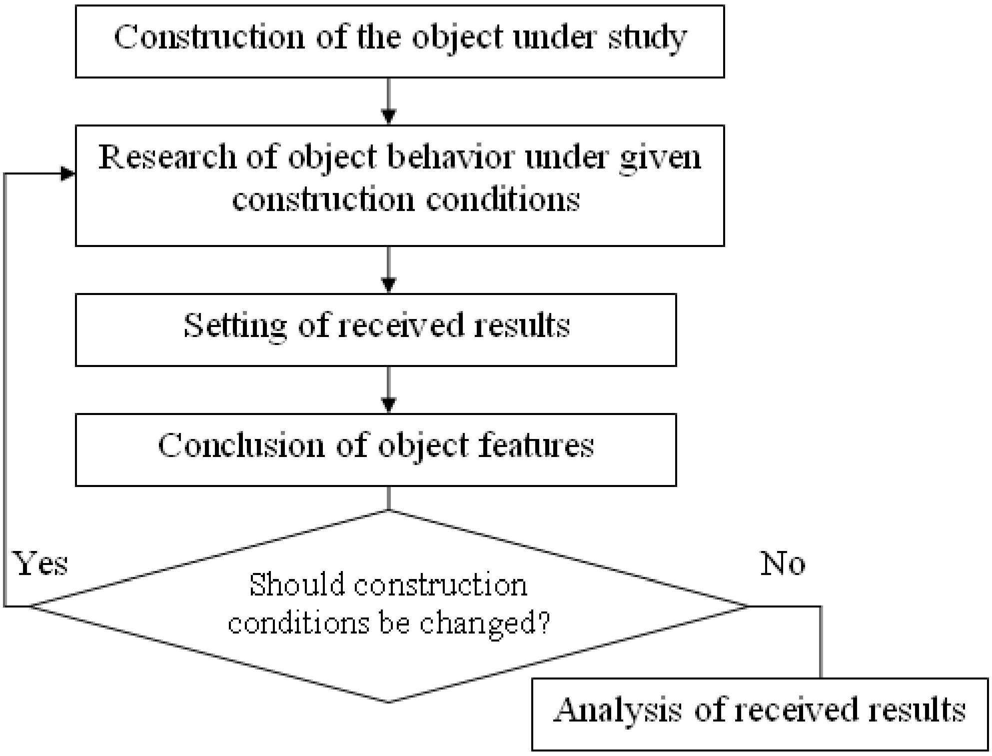

Fig. 1 Computer experiment

2. Materials and methods

Target of research: to demonstrate the experimental capabilities of Dynamic Geometry Environment GeoGebra. Main sources were the journal publications and GeoGebra software.

During the experimental work on the issue of the article were used: theoretical methods (collection and analysis of material on a research problem); experimental methods (the computer experiment).

3.Discussion

Computer experiment conducted by means of Dynamic Geometry Environment is a kind of model experiment in which a dynamic drawing (model) of a geometric configuration occur as an object of study [13, c. 42]. The ability to apply not only traditional instruments, (a ruler, a compasses, a protractor, etc.) in mathematical practice but also to use universal or special purpose software is one of the important instrumental research skills of students.

The didactic possibilities of Dynamic Geometry Environment GeoGebra are described in details in works [14, c. 117; 15, c. 46; 16, c. 561]. Let us consider the methodical and graphical capabilities of the system from the point of view of realization of a computer experiment.

The list of computer tools in Dynamic Geometry Environment GeoGebra includes:

– a standard set of tools that allows to create basic geometric objects such as a point, a line, a circle, a vector, a polygon, an angle;

– a set of tools that perform additional operations on geometric objects (dividing a segment in half, dividing an angle into *n* of equal parts and others);

– tools that allow performing experimental and research work (measuring the length of a segment,

– measuring the size of an angle, etc.);

– input of data, commands and functions in the input line;

– ability to create animations - automatic moving points along specified trajectories.

We will separately consider the tools which can be used to implement the tasks of the computer experiment (Fig. 2):

– tools for detecting the metric and positional features of the object («Distance or length», «Angle», «Area», «Straight line inclination»);

– tools for obtaining data about the ratio of metric and positional features of objects («Relation of objects»);

– the possibility of a moving point to leave a trace (feature of the object «Leave a trace»);

– a tool for parametric definition of changes in a quantity («Slider»);

– creation of experimental data tables (menu command««Type»→«Table», tool «Record in the table»);

– creation of dynamic texts («Text»).

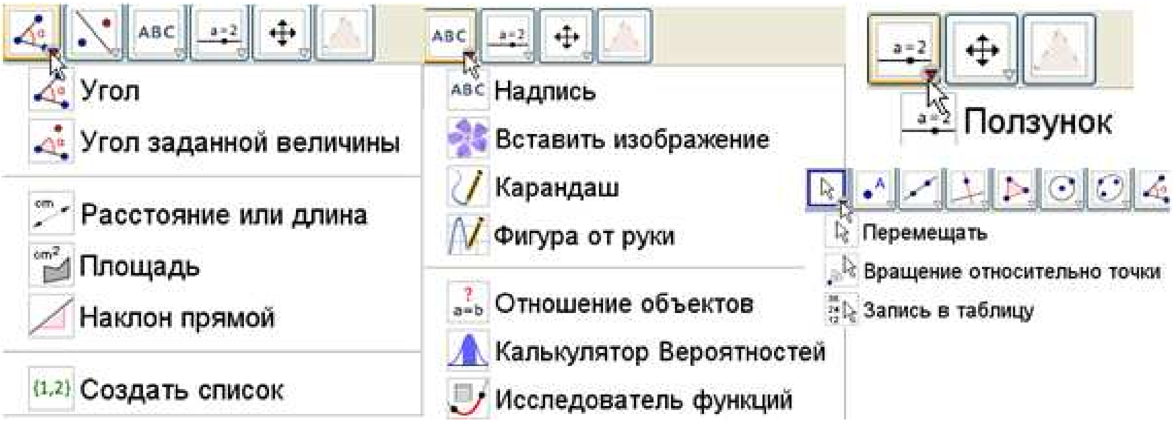

Fig. 2 Tools for a computer experiment

Let us demonstrate the usage of tools to identify the metric and positional features of the object («Distance or length», «Angle»), a tool for obtaining information about the relationship between metric and positional features of objects («Relation of objects») and the creation of dynamic texts «Text» for the construction of the center of the circle circumscribed about the triangle .

The theorem (about one of the remarkable points of a triangle). The middle perpendiculars to the sides of the triangle (the mediatrix) are crossed at one point which is the center of the circle circumscribed about the triangle [17, c. 178].

The purpose of the construction: an experimental verification of the predicating that the middle perpendiculars to the sides of the triangle are crossed at one point which is the center of the circle circumscribed about the triangle.

Algorithm for constructing a dynamic drawing (Fig. 3):

№	Steps of construction	Computer tools
1.	Make an arbitrary the triangle ABC	«Polygon»
2.	Make the middle perpendiculars to the sides of the triangle. Mark crossing points of the middle perpendiculars O as the intersection of two objects. To rename points, use the feature of the object «Rename»	«The median perpendicular» «Intersection of two objects»
3.	Mark crossing points of middle perpendiculars with the sides of the triangle R, S, P. Construct the segments OR, OS, OP	«Intersection of two objects» «Two-point segment»
4.	Check properties of the middle perpendicular constructed to the sides of the triangle	«Distance or length» «Angle»
5.	Check that the point O belongs to the middle perpendiculars Hide perpendicular lines using the object property «Show the object»	«Relation of objects»
6.	Measure the lengths of the segments OA, OB, OC. Check the equality of segments	«Distance or length» «Relation of objects»

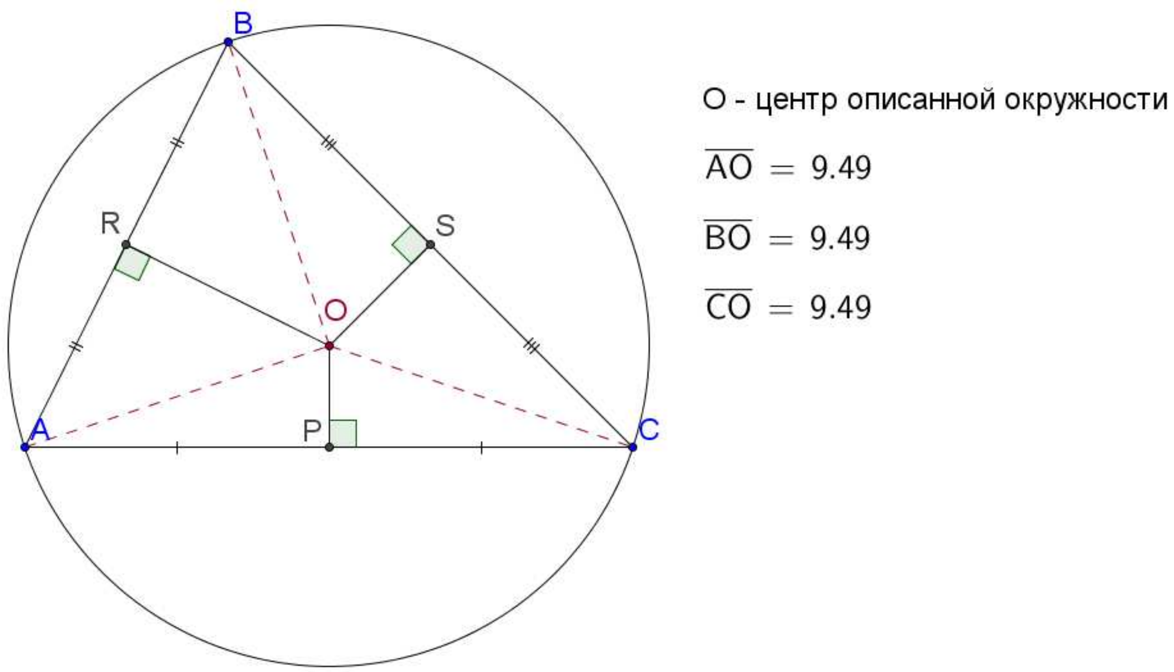

Fig. 3 The construction of the triangle's mid-perpendicular and the circle circumscribed to it

For the experimental verification of the statement we use the following GeoGebra tools:

– tools for detection metric and positional features of the object («Distance or length», «Angle»);

– tools for obtaining information about the ratio of metric and positional features of objects («Relation of objects»);

– creation of dynamic texts («Text»).

Usage of the tool «Relation of objects» allows us to evaluate the results of a geometric construction: belonging of the point to the pointed segment or the line, equality of the lengths of two segments (the segments will not be identical because of their different locations) (Fig. 4). Dynamic text displays the characteristics of the examined drawing objects.

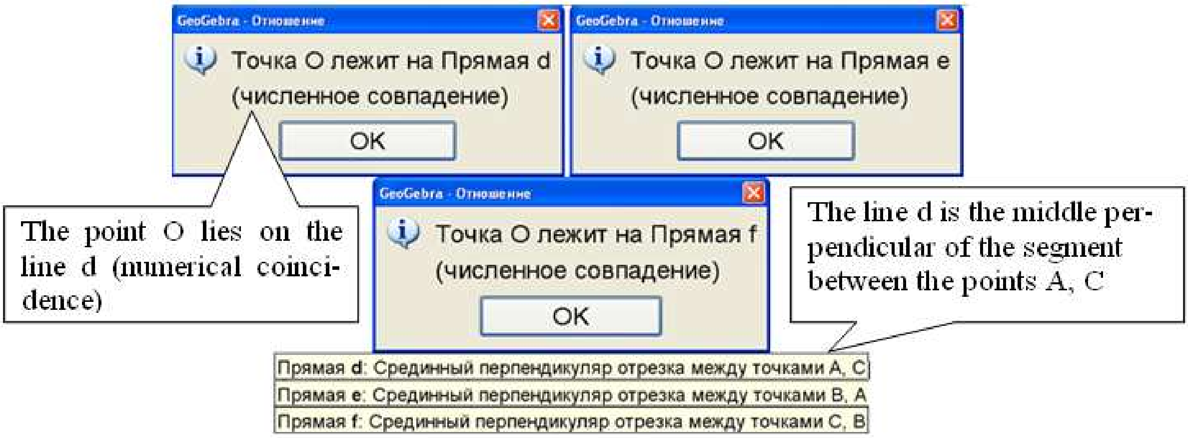

Fig. 4 Check that the point O belongs to the middle perpendiculars

From the history of mathematics it is known that many mathematical results were obtained during the research, through experiments and inductive reasoning and only later they were proved by a deductive method.

For creation of the dynamic drawings saving an algorithm of creation of geometrical models in case of the changing values of one or several values the Slider tool is used.

Slider is a tool that contains a point-slider freely movable along some line. A value is associated with this point, which is used as a parameter. In case of relocation of an engine of the slider from smaller value to bigger it is also back possible to watch behind change of properties of the studied object depending on a parameter value.

The characteristics of the tool «Slider» are: the parameter type (Number or Angle*)*, the parameter name, the minimum and maximum value of the parameter (the range of allowed values), the step of the parameter change (Fig. 5).

Additional characteristics of the tool «Slider» can be defined in the tab Slider: the variant of its location (horizontal or vertical), the prohibition or permission to move it in the graphics window with the help of the mouse («checkbox» fixed), the size (the Width window). The animation capabilities of the tool «Slider» are defined in the Animation tab.

After determining all the characteristics, a slider image appears in the graphics window and in the object panel appears a free object (name and value of the parameter).

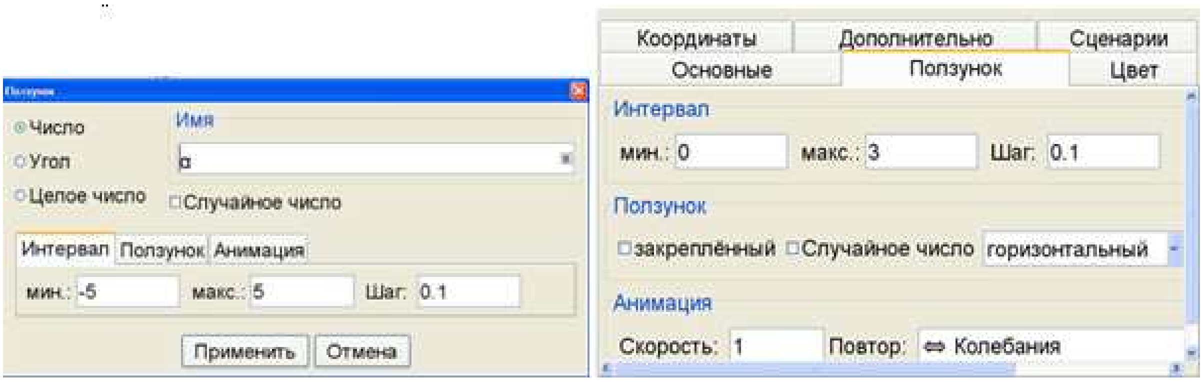

Fig. **5** Characteristics of the Slider tool

The Slider tool is used to perform research work with the following tools:

- a segment of a given length (one of the ends of the segment and its length – parameter are specified);

- the angle of the set value (one of the sides of the angle, the vertex of the angle and the value of the angle – parameter are specified);

- -the circle on the center and a radius (the center of the circle and a radius – parameter are specified);

- the homothety about the point (the projected object, the center and a homothetic coefficient – parameter are specified);

- -the turn around the point on an angle (the source object, the center of rotation and the angle of rotation – parameter are specified).

The possibilities of the Slider tool will be demonstrated on the example of Menelaus' theorem presented in the book «Spheres» which was written by the Greek mathematician and astronomer Menelaus of Alexandria (about 100 AD). This theorem is about the simplest of polygons, about a triangle that attracted attention of both ancient scientists (Geron, Menelaus, Ptolemy) and scientists closer to our time (Euler, Poncelet, etc.). With the help of it is easy and elegant to solve a whole class of problems.

The Menelaus theorem is presented in Chapter VIII «Some Information from Planimetry» of a textbook for the 10^{th}-11^{th} grade [18, c. 200], which is not mandatory for the basic level of preparation but necessary for solving the problems of planimetry of the profile level in a rather complex form, but with mathematical precision and completeness. We will consider it in a more accessible form, conduct a computer experiment.

Theorem 1. If the points C_1 and A_1 are taken on the sides AB and BC of the triangle ABC respectively, and the point B_1 is taken on the extension of the side of the AC, then the points C_1, A_1 and B_1 lie on the same line then and only then the equality is performed:

$$\frac{AC_1}{C_1B} \cdot \frac{BA_1}{A_1C} \cdot \frac{BB_1}{B_1A} = 1$$

In a more accessible form: when composing the Menelaus equality one make a detour of the triangle in any direction, passing from the vertex of the triangle to the other peak vertex through the point of intersection of the cutting line with the side or its continuation, completing the detour at the top from which the movement was started (Fig. 6).

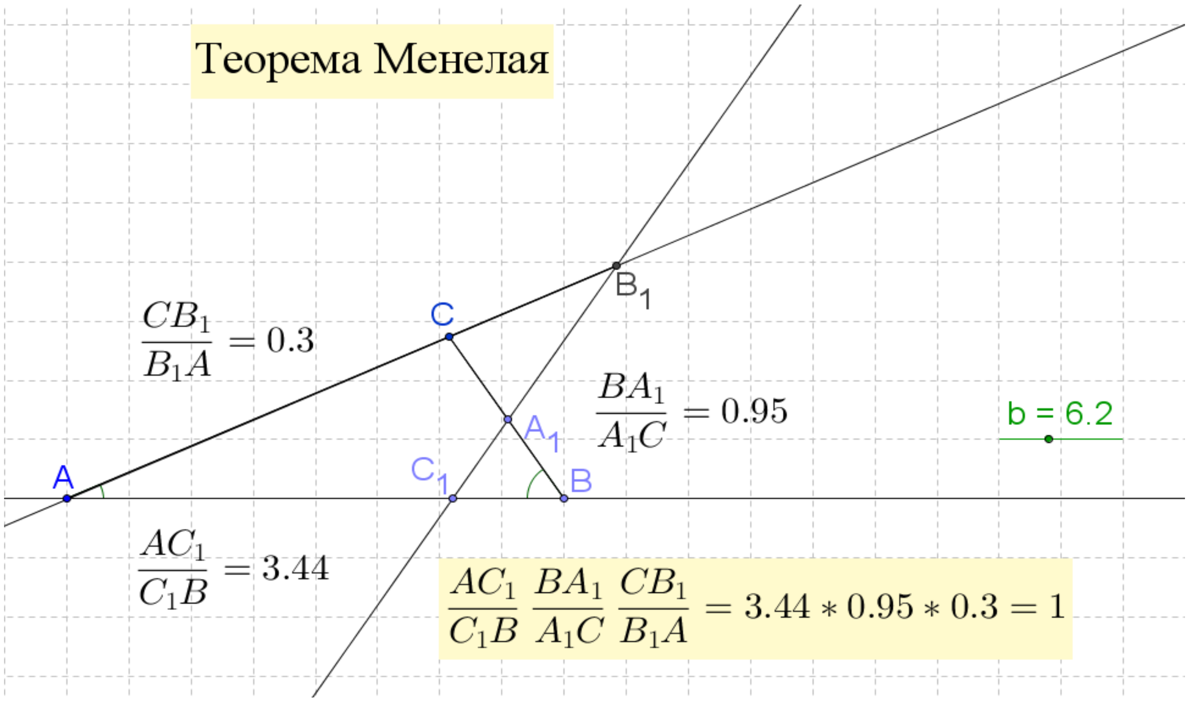

Fig. 6 Menelaus theorem

To open the relationship of the theorem we try to use GeoGebra instead of using traditional tools (a pencil, a ruler, a compasses). To verify the truth of the relation under study, using the «Move» tool for point C_1 (or/and point A_1), we observe the changes in the numerical characteristics of the objects of the geometric construction having built, while preserving the basic algorithmic features of the sketch with the help of the created dynamic text, for the output of which we use the «Inscription» tool. We come to the conclusion: changes in the characteristics of the drawing objects (lengths of the resulting segments) do not affect the value of the theorem: it remains unchanged and equal to 1.

If it is necessary to record the observed changes in the experimental data table, one can use the «Slider» tool to change the length of the segment AC_1, which will cause the lengths change of the segments C_1B, CB_1, B_1A (Fig. 7).

	A	B	C	D	E
1	AC_1	$k_1 = AC_1/C_1B$	$k_2 = BA_1/A_1C$	$k_3 = CB_1/B_1A$	$k_1 * k_2 * k_3$
2	5	2.11	0.95	0.63	1
3	5.1	1.76	0.95	0.6	1
4	5.2	1.86	0.95	0.56	1
5	5.3	1.96	0.95	0.53	1
6	5.4	2.08	0.95	0.5	1
7	5.5	2.2	0.95	0.48	1
8	5.6	2.33	0.95	0.45	1
9	5.7	2.48	0.95	0.42	1
10	5.8	2.64	0.95	0.4	1
11	5.9	2.81	0.95	0.37	1
12	6	3	0.95	0.35	1

Fig. 7 Experimental Data Table

If it is necessary to fix data on the observed changes in the experimental data table, then we use the GeoGebra capabilities to collect the experimental data, that is the experimental data table that appears in the graphics window, when you select the «View»→«Table» command.

With the help of the «Write to the Table» tool the studied drawing objects as well as the expressions in which they can be used, are sequentially tabulated into the columns, what is more the variable, on which characteristics of the objects under study depend, is tabulated into the first column.

Filling in the experimental data table for the investigated ratios of the segments and their product is carried out for variable values of the length of the segment AC_1 in the setting of fixed values of the lengths of the segments BA_1 and A_1C. It is recommended that the study should be carried out more than once to verify the truth of the statement.

To demonstrate the proof of the Menelaus theorem one can create a step-by-step proof in the form of a «live drawing» on a single canvas with a sketch using the «Slider» tool (Fig. 8).

Fig. 8 Menelaus Theorem Proof

4.Conclusion

Widespread use in the practice of teaching computer technology mathematics allows making the process of computer experimentation less time consuming and more exciting.

GeoGebra's freely-distributed dynamic geometry environment, which has a Russian version and a friendly interface that can be conveniently used both in classrooms and at home, is chosen as a computer technology for implementing a computer experiment.

The presented capabilities of the GeoGebra system can be used to conduct a computer experiment with the aim of discovering new knowledge, hypotheses testing, involving students in the active process of learning new material. The obtained results of computer experiments give an idea of the dynamic geometry environment GeoGebra as a computer technology that will bring visibility and thrill to the practice of teaching mathematics.

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