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О МАТЕМАТИЧЕСКОЙ МОДЕЛИ ВЛИЯНИЯ НЕСОБЛЮДЕНИЯ ПРОФИЛАКТИЧЕСКИХ МЕР ПО ПРЕДУПРЕЖДЕНИЮ РАСПРОСТРАНЕНИЯ ВИЧ/СПИДА СРЕДИ ГЕТЕРОГЕННОГО НАСЕЛЕНИЯ

Басси Басси Эченг
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В данной работе рассматривается математическая модель влияния несоблюдения профилактике ВИЧ/СПИДа среди гетерогенного населения, основанная на известную модель Kimbir et al (2006). Эффективность использования презервативов и последствия несоблюдения населением с профилактической меры (презерватив) являются целью данной научной работы. В этой работе, с определенными коэффициентами, нелинейных используется модель, которая состоит из системы шести дифференциальных уравнений для различных групп населения (шести группам населения) для получения модельных уравнений. По сравнению с существующей моделью Kimbir, предлагаемая модель с большой степени учитывает рождаемость изучаемого населения. Численное моделирование уравнений модели показывает, что сокращения скорости передачи ВИЧ/СПИДа могут быть эффективно достигнуты в течение определенного времени, и только там, где сравнительно высокая степень презерватив эффективность и высокий уровень соблюдения по восприимчивы и зараженным наблюдаются. Из полученных результатов мы видим, что контроль ВИЧ/СПИДа в гетеросексуальной популяции зависит от чистой соответствие и эффективность рекомендованных профилактики (использование презервативов). В качестве рекомендации, модель ориентирована на интенсивное обучение и продолжающейся кампании по повышению информированности населения со стороны правительственных и неправительственных учреждений по эффективному использованию презерватива

Ключевые слова: ВИЧ / СПИД, ПРАКТИКА

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Mathematical and Computer Sciences

ON MATHEMATICAL MODEL OF THE IMPACT OF NON-COMPLIANCE WITH PREVENTIVE MEASURES FOR THE PREVENTION OF THE SPREAD OF HIV/AIDS AMONG HETEROGENEOUS POPULATION

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In this article we consider a mathematical model of effect of non-compliance with the prevention of HIV/AIDS among a heterogeneous population based on known model by Kimbir et al (2006). The effectiveness of a condom use and implications of non-compliance with a population of preventive measures (condoms) are the aim of this research work. In this work, with definite coefficients, nonlinear model is used, which consists of system of six differential equations for different population groups (six groups of the population) to obtain the model equations. Compared with the existing model by Kimbir, the proposed model to a large extent, takes into account the birth rate of the studied population. Numerical simulation of the model equations shows that reducing the rate of transmission of HIV/AIDS can be effectively achieved within a certain time, and only where relatively high condom efficacy and high compliance by susceptible and infected are observed. From the obtained results, we can see that the control of HIV/AIDS in the heterosexual population depends on the net compliance and effectiveness of the recommended prevention (condom use). As a recommendation, the model focuses on intensive training and ongoing campaigns to raise the awareness of the population by governmental and non-governmental agencies on the effective use of the condom

Keywords: HIV/AIDS, CONDOM USE, NON-

ИСПОЛЬЗОВАНИЯ ПРЕЗЕРВАТИВОВ, НЕСОБЛЮДЕНИЕ, ЭФФЕКТИВНОСТЬ, ГЕТЕРОГЕННЫЙ, ВОЗДЕЙСТВИЕ, ТРАНСМУТИРОВАТЬ

COMPLIANCE, EFFICACY, HETEROGENEOUS, IMPACT, TRANSMUTE

1. INTRODUCTION

The basic routes of the Human Immune Deficiency Virus (HIV) transmission between persons are now well understood and widely known, but the non-adherent to the application of recommended preventive measures which leads to HIV prevalence and trends among population has remained an area of debate and scientific research [1]. Diseases can be transmitted either horizontally or vertically. In the case of HIV, horizontal transmission occurs from direct contact between an infected individual and susceptible persons. Vertical transmission is as a result of direct transfer from mother-to-child or newborn offspring [2], as over 40% of all HIV cases result from mother-to-child transmission.

The Human Immune Deficiency Virus (HIV) is known to be the causative agent of the deadly disease called the Acquired Immune Deficiency Syndrome (AIDS), the extensive spread of which appears to have commenced in the early 1980s, [3]. In sub-Saharan Africa, over 2.5 million children under the age of 15, have died of AIDS as a result of been exposed to HIV during labor or breastfeeding. Thus, the impact of HIV transmission has been felt mainly in Africa [4], where the level of literacy is low, the poverty level is very high and the quality of health services is generally very poor.

The dynamics of HIV/AIDS has moved beyond the virus and risk factors associated with its transmission to a more detailed understanding of the mechanisms associated with the spread, distribution and impact of any intervention on the population [5]. Transmission modes of HIV/AIDS are basically via heterosexual contacts, blood transfusion, and contaminated injecting equipment [6].

Due to non-availability of any known medical cure for HIV/AIDS, therapeutic treatment strategies appear promising for retarding the progression of HIV-related diseases. In other words, prevention has remained the most effective strategy against the HIV/AIDS epidemics [7]. Control and intervention programs are focused towards educating the people on behavioral change. Other method of control include barrier contraceptives (i.e. condom use), which is a single strategy in the prevention of HIV/AIDS. Condom use, as a strategy to halt the HIV epidemics is considered in terms of its efficacy and compliance. The study of these two factors has been more rewarding through mathematical modeling.

Mathematical models of the transmission of HIV have proven to be useful in providing a logical structure within which to incorporate knowledge and test assumptions about the complex HIV epidemic [8]. Through mathematical modeling, the control of HIV/AIDS have been formulated as far back as 1987, when [9], developed simple function for the growth in the number of individuals who will develop AIDS and for the distribution of incubation period of those individuals [10].

Other models for the control of HIV epidemic include the following aspects: age-structure population, differential infectivity and stage progression; fully-integrated immune response model (FIRM), defense model, random screening, contact tracing, use of condom etc., for example, see [5,8,10,11,12, 13,14,15,16,17].

The motivating factor of this present work lies in the model [10], on a two-sex mathematical model for the prevention of the spread of HIV/AIDS in a varying population. In that study, the preventive measure (condom use) was, as well, introduced to the susceptible males among others. The model was aimed at studying the efficacy of the preventive measure which consequently leads to reduction of HIV infection within a given period of time. The impact or consequences of non-adherent to the usage of the preventive measure were conspicuously not treated.

New cases of HIV infection due to unprotected sex [18], indicate a high level of non-compliance in the application of preventive measure by both susceptible and infected individuals. In this present study, which is an extended version of the old model [10], we have developed a model for transmission dynamics of HIV/AIDS in a population of varying size with vertical and other demographical and epidemiological factors. Our purpose is to formulate a model for AIDS epidemic control measure that unveil and could transmute the impact of non-compliance and the trend of the efficacy of condom use in either horizontal or vertical HIV transmission. This work is devoted to the study of the impact of non-compliance by the population.

Consequently, in section 2, with improved coefficients and addition of novel features, we formulate a non-linear system of ordinary differential equations that model not only the efficacy of the condom but that which takes into account, the dynamics of non-compliance in the application of the preventive measure by both susceptible and infected males in a varying population. A transformation of the model equations into non-dimensionless proportions is carried out on section 3, followed by numerical simulation of the transformed equations in section 4. The last section covers a number of discussions, concluding remarks and recommendations.

2. THE MODEL EQUATION

In formulating the model equations, we consider figure 1(a & b) below:

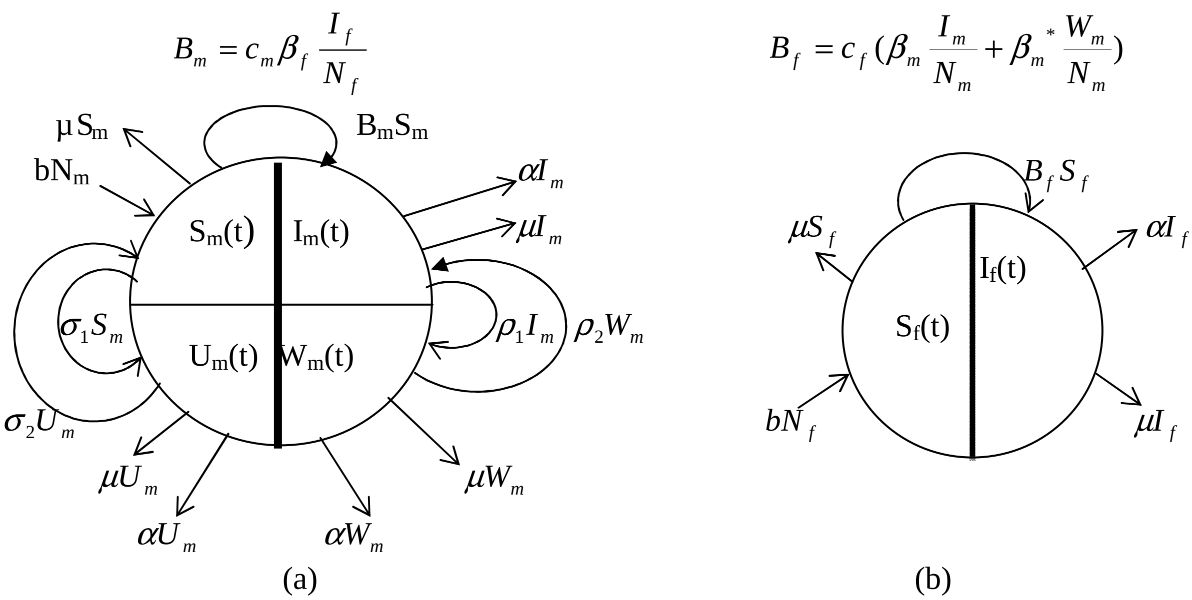

Fig. 1(a & b): A flow-chat of the model for male and female population for the prevention of HIV/AIDS in different groups of the population.

From figure 1(a), the functions (parameters) used in the model for the male populations are as defined below:

$S_m(t)$	-	Number of susceptible males at time t;
$I_m(t)$	-	Number of infected males at time t;
$W_m(t)$	-	Number of infected males who use the condom at time t;
$U_m(t)$	-	Number of susceptible males who use the condom at time t;
$N_m(t)$	-	Total population of males at time t;
$B_m(t)$	-	The rate at which males are infected per unit time (incident rate);
bN_m	-	Natural birth rate of male population, $b \geq 0$;
μ	-	Natural death rate, $\mu \geq 0$;
α	-	AIDS – related death rate, $\alpha \geq 0$;
ρ_1	-	The proportion of infected males who use the condom per unit time;
ρ_2	-	The proportion of infected males who initially use the condom and then decline per unit time;
σ_1	-	The proportion of susceptible males who use the condom per unit time;
σ_2	-	The proportion of susceptible males who initially use the condom and then decline per unit time;
c_m	-	Average number of contacts by males with females per unit time;
β_m	-	Probability of transmission by an infected males; and
β_m^*	-	Probability of transmission by an infected males who use the condom.

Where

$$N_m(t) = S_m(t) + I_m(t) + W_m(t) + U_m(t)$$

Using figure (1b), we also derive the functions (parameters) used in the model for the female population:

$S_f(t)$	-	Number of susceptible females at time t;
$I_f(t)$	-	Number of infected females at time t;
$N_f(t)$	-	Total population of females at time t;
bN_f	-	Natural birth rate of female population, $b \geq 0$;
$B_f(t)$	-	The rate at which females are infected per unit time (incident rate);
μ	-	Natural death rate, $\mu \geq 0$;
α	-	AIDS – related death rate, $\alpha \geq 0$;
c_f	-	Average number of contacts by females with males per unit time; and
β_f	-	Probability of transmission by an infected females.

Where

$$N_f(t) = S_f(t) + I_f(t)$$

The assumptions in this model follow from those made in the previous model [15], except that here, we assume that:

(i) susceptible individuals also use the condom, with a varying degree of efficacy;

(ii) a number of susceptible males that use the condom initially, decline thereafter, per unit time;

(iii) infected males who use the condom initially, decline thereafter per unit time; and

(iv) the impact of b, (natural birth rate), is only in the susceptible population.

In constructing the model, coefficients are considered positive, and the signs before the terms are taken with a plus if the arrow enters the area and with a minus sign, if the arrow goes out of scope. As can be seen, the model takes into account the birth rate b, natural population decline μ and death from disease, α.. In this study, from the flow-chat (fig.1 (a)), the model is governed by the following systems of non-linear ordinary differential equations:

$$S_m' = bN_m N_f - B_m S_m - \mu S_m - \sigma_1 S_m + \sigma_2 U_m; \quad (2.1)$$

$$I_m' = B_m S_m + \rho_2 W_m - (\mu + \alpha + \rho_1) I_m; \quad (2.2)$$

$$W_m' = \rho_1 I_m - (\rho_2 + \mu + \alpha) W_m; \quad (2.3)$$

$$U_m' = \sigma_1 S_m - (\sigma_2 + \mu) U_m; \quad (2.4)$$

From figure 1 (1b), we also derive the functions (parameters) used in the model for the female population:

$$S_f' = bN_f N_m - B_f S_f - \mu S_f \quad (2.5)$$

$$I_f' = B_f S_f - (\mu + \alpha) I_f \quad (2.6)$$

Differential equations is supplemented by algebraic equations

$$N_m = S_m + I_m + W_m + U_m \quad (2.7)$$

$$N_f = S_f + I_f \quad (2.8)$$

The coefficients B_m and B_f are expressed in terms of the relative $\frac{I_f}{N_f}$ and $\frac{I_m}{N_m}$, $\frac{W_m}{N_m}$ of HIV-infected individuals:

$$B_m = c_m \beta_f \frac{I_f}{N_f} \quad (2.9)$$

$$B_f = c_f (\beta_m \frac{I_m}{N_m} + \beta_m{}^* \frac{W_m}{N_m}) \quad (2.10)$$

If $\rho_1 = \rho$; $\sigma_1 = \sigma$; ρ_2, σ_2, and β_m* are zero, then we obtain the model as in [15] and if only $\rho_1 = \rho$; $\sigma_1 = \sigma$; ρ_2, σ_2 are zero with $\beta_m^* \neq 0$, we obtain the model as in [10]. Therefore, these parameters and the present equations (2.4) and (2.10), make the difference between models [10, 15] and the new model. The detail parameters are summarized as in table 1, below:

Table 1: The original data are summarized in tabular format:

Symbols	Expression for the males	Symbols	Expression for the females
S_m	number of Susceptible males at time *t*;	S_f	number of susceptible females at time *t*;
I_m	number of infected males at time *t*;	I_f	number of infected females at time t;
W_m	number of infected males who use the condom at time t;	N_f	Sf(t) + If(t) - total population of females at time t;
U_m	number of susceptible males who use the condom at time t;	bN_f	natural birth rate of females population, $b \geq 0$;
N_m	Sm(t) + Im(t) + Wm(t) + Um(t) - total population of males at time t;	B_f	the rate at which females are infected per unit time (incident rate);
B_m	the rate at which males are infected per unit time (incident rate);	μ	natural death rate, $\mu \geq 0$;
bN_m	natural birth rate of male population, $b \geq 0$;	α	AIDS – related death rate, $\alpha \geq 0$;
μ	natural death rate, $\mu \geq 0$;	c_f	average number of contacts by females with males per unit time;
α	AIDS – related death rate, $\alpha \geq 0$;	β_f	Probability of transmission by an infected female.
ρ_1	the proportion of infected males who use the condom per unit time;		
ρ_2	the proportion of infected males who initially use the condom and then decline per unit time;		
σ_1	the proportion of susceptible males who use the condom per unit time;		
σ_2	the proportion of susceptible males who initially use the condom and then decline per unit time;		
c_m	average number of contacts by males with females per unit time;		
β_m	Probability of transmission by an infected males;		
β_m*	Probability of transmission by an infected male who use the condom.		

3. TRANSFORMATION OF MODEL EQUATIONS TO NON-DIMENSIONAL FORM

It is essential that for easy handling and to initiate the biological meaning of the proportions of infected individuals, which defines the prevalence of infection, we transform the model equations into proportions.

Let,

$N_m(0) = N_m, N_f(0) = N_f$ and $m(t) = N_m.N_f, f(t) = N_f.N_m$

Then,

$$s_m = S_m / N_m, \tag{3.1}$$

$$s_f = S_f / N_f, \tag{3.2}$$

$$y_m = I_m / N_m, \tag{3.3}$$

$$y_f = I_f / N_f, \tag{3.4}$$

$$u_m = U_m / N_m, \tag{3.5}$$

$$w_m = W_m / N_m \tag{3.6}$$

Clearly, in terms of proportion,

$$m(t) = N_m(t) / N_m = s_m + y_m + w_m + u_m \tag{3.7}$$

and

$$f(t) = N_f(t) / N_f = s_f + y_f \tag{3.8}$$

The coefficients (2.9) – (2.10), in the dimensionless form are obtain as

$$B_m = c_m \beta_f y_f / f(t) \tag{3.9}$$

$$B_f = c_f(\beta_m y_m + \beta_m{}^* w_m) / m(t) \tag{3.10}$$

Then equations (2.1) – (2.4) can be rewritten as

$$s_m' = bm(t) - B_m s_m - \mu s_m - \sigma_1 s_m + \sigma_2 u_m; \tag{3.11}$$

$$y_m' = B_m s_m + \rho_2 w_m - (\mu + \alpha + \rho_1) y_m; \tag{3.12}$$

$$w_m' = \rho_1 y_m - (\rho_2 + \mu + \alpha) w_m; \tag{3.13}$$

$$u_m' = \sigma_1 s_m - (\sigma_2 + \mu) u_m \tag{3.14}$$

Similarly, equations (2.5) and (2.6) can as well be rewritten as

$$s_f' = bf(t) - B_f s_f - \mu s_f \quad (3.15)$$

$$y_f' = B_f s_f - (\mu + \alpha) y_f \quad (3.16)$$

Thus, the changes in the various groups of the population are discussed on the basis of equations (3.11) - (3.16) as summarized below:

Table 2: Proportions of the dynamics of HIV transmission

Group	**Derivatives**	*Eqn. no.*
S_m	$s_m' = bm(t) - B_m s_m - \mu s_m - \sigma_1 s_m + \sigma_2 u_m$	(3.11)
I_m	$y_m' = B_m s_m + \rho_2 w_m - (\mu + \alpha + \rho_1) y_m$	(3.12)
W_m	$w_m' = \rho_1 y_m - (\rho_2 + \mu + \alpha) w_m$;	(3.13)
U_m	$u_m' = \sigma_1 s_m - (\sigma_2 + \mu) u_m$	(3.14)
S_f	$s_f' = bf(t) - B_f s_f - \mu s_f$	(3.15)
I_f	$y_f' = B_f s_f - (\mu + \alpha) y_f$	(3.16)

4. NUMERICAL SIMULATIONS AND ANALYSIS OF THE MODEL

The model represents systems of 6 differential equations concerning 6 considered groups of the population, $s_m, y_m, w_m, u_m, s_f, y_f$ with initial values and parameters given in the table 3.

TABLE 3: Table of parameter values for Variants 1 -5

Variant	b	α	μ	$\sigma 1$	$\sigma 2$	$\rho 1$	$\rho 2$	c_m	c_f	β_m^*	β_m	β_f
1(*)	0.02	0.1	0	0.5	0	0.9	0	5	5	0.1	0.14	0.14
2	0.02	0.04	0	0	0	0	0	5	5	0.1	0.1	0.2
3	0.02	0.04	0.1	0	0	0	0	5	5	0.1	0.1	0.2
4	0.02	0.04	0.1	0.2	0	0.2	0	5	5	0.1	0.1	0.2
5	0,02	0,04	0.1	0.2	0.1	0.2	0.1	5	5	0.1	0.1	0.2

$V_2 - s_m - S_m$: $s_m(0) = 0.7$
$V_3 - y_m - I_m$: $y_m(0) = 0.1$
$V_4 - u_m - U_m$: $u_m(0) = 0.1$
$V_5 - w_m - W_m$: $w_m(0) = 0.1$
$V_6 - s_f - S_f$: $s_f(0) = 0.8$
$V_7 - y_f - I_f$: $y_f(0) = 0.2$

Where initial valves of all characteristics

$$\begin{aligned} &V_2 - s_m - S_m,\ s_m(0) = 0.7 \\ &V_3 - y_m - I_m,\ y_m(0) = 0.1 \\ &V_4 - u_m - U_m, u_m(0) = 0.1 \\ &V_5 - w_m - W_m, w_m(0) = 0.1 \\ &V_6 - s_f - S_f,\ s_f(0) = 0.8 \\ &V_7 - y_f - I_f, y_f(0) = 0.2 \end{aligned}$$

are constant.

In the program we shall designate $s_m, y_m, w_m, u_m, s_f, y_f$ through a vector $Y = \{Y_1, Y_2, Y_3, Y_4, Y_5, Y_6\}^T \Rightarrow Y = \{Y_i\}_{i=1}^{6}$ э $Y_m = \sum_{i=1}^{4} Y_i$ and $Y_f = \sum_{i=5}^{6} Y_i$.

Then the system of differential equations can be written as

$$\begin{cases} \dfrac{dY}{dt} = f(Y),\ t \in [0,T] \\ Y(0) = Y_0 \end{cases}.$$

To integrate the system, one can use different numerical methods. The simplest, is the Euler's first order of accuracy. However, in MATHCAD, there is built-in function "rkfixed" realizing Runge-Kutter method of accuracy of order 4.

Importantly, it is essential to note that in analyzing the results of the computer simulations that follows, our interest is in the corresponding changes that are likely to occur in proportions of infected males and females (y_m, y_f), for each of variant (1-5), as in table 3. The efficacy of the condom are determined by the behavior of σ_1, ρ_1 and $\beta^*{}_m$ while the non-compliance situation are determined by σ_2 and ρ_2 respectively. These do not mean a play-down on other parametric factors. So that if $V^{\langle 1 \rangle}$, represent the time interval *t*, (in years) and from Table 2, letting $V^{\langle 2 \rangle}, V^{\langle 3 \rangle}, V^{\langle 4 \rangle}$, $V^{\langle 5 \rangle}, V^{\langle 6 \rangle}$ and $V^{\langle 7 \rangle}$,equal $s_m, y_m, w_m, u_m, s_f, y_f$ and y_f' respectively, then for $\beta_m^* = \beta_m 1$, the corresponding graphs obtained from Table 3, are represented as in figures (4.1) – (4.5), below:

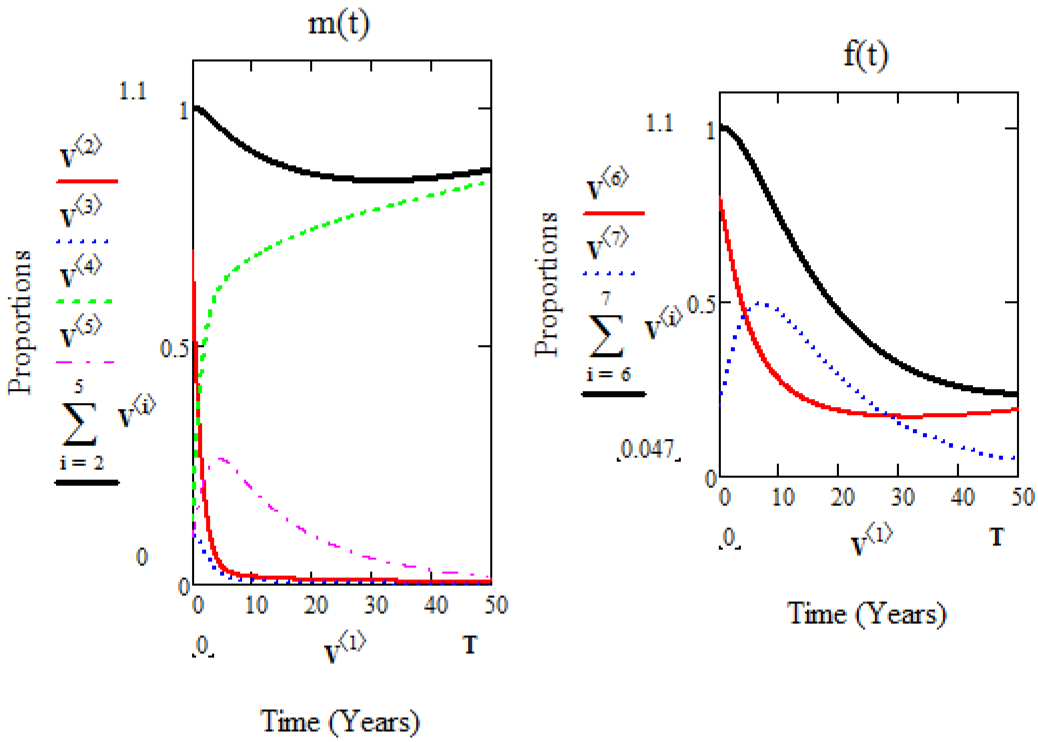

Fig.4.1: Graphical simulations of s_m, y_m, w_m, u_m, s_f and y_f from model (3.11) – (3.16) against time with $V^{\langle 1 \rangle}$ - the sum of male and female proportions. Parameter values are in variant (1) - Table 3.

From figure 4.1 above, we investigate the simulation of the old model [10], where natural birth rate were not defined in relation to the existence of interaction between the male and female population as presented by equations (2,1) and (2.5) of the present model. For compatibility, the only variation in that model is the birth rate (i.e. $b = 0.02, \neq 0.5$). With high average number of contacts by both sexes (c_m, c_f); $50\% - 90\%$ condom use and compliance level not accounted for (i.e. $\sigma_2 = 0 = \rho_2$), it is seen that the entire susceptible became infected within 10years, leading to rapid extinction of infected (without condom use) population within 20 - 30 years. Prolonged lifespan and increase in population were experience by the proportion of the males that used the condom. The entire female population was seen contaminated by the infection, leading to slow death after 50 years. The result indicates that condom use was not efficacious and coupled with non-compliance situation.

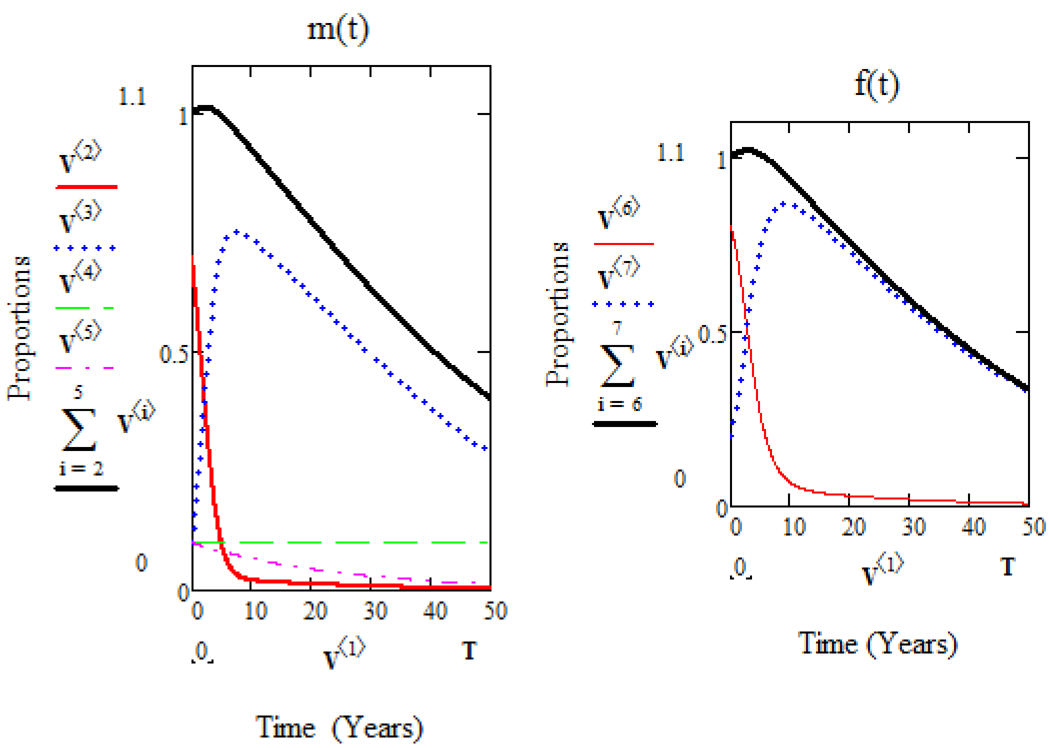

Fig.4.2: Graphical simulations of s_m, y_m, w_m, u_m, s_f and y_f from model (3.11) – (3.16) against time with $V^{\langle i \rangle}$ - the sum of male and female proportions. Parameter values are in variant (2) - Table 3.

From fig. 4.2 above, we studied the effect of the factors: $\alpha = 0.04, b = 0,02, \beta_m 1 = 0.1, \beta_m = 0.1, \beta_f = 0.2$. We observed initial increase in infection by both male and female infected population, consuming the susceptible population after an average of 30 - 40 years. With no preventive measure (condom use), the situation is seen to graduate to slow death of the infected population.

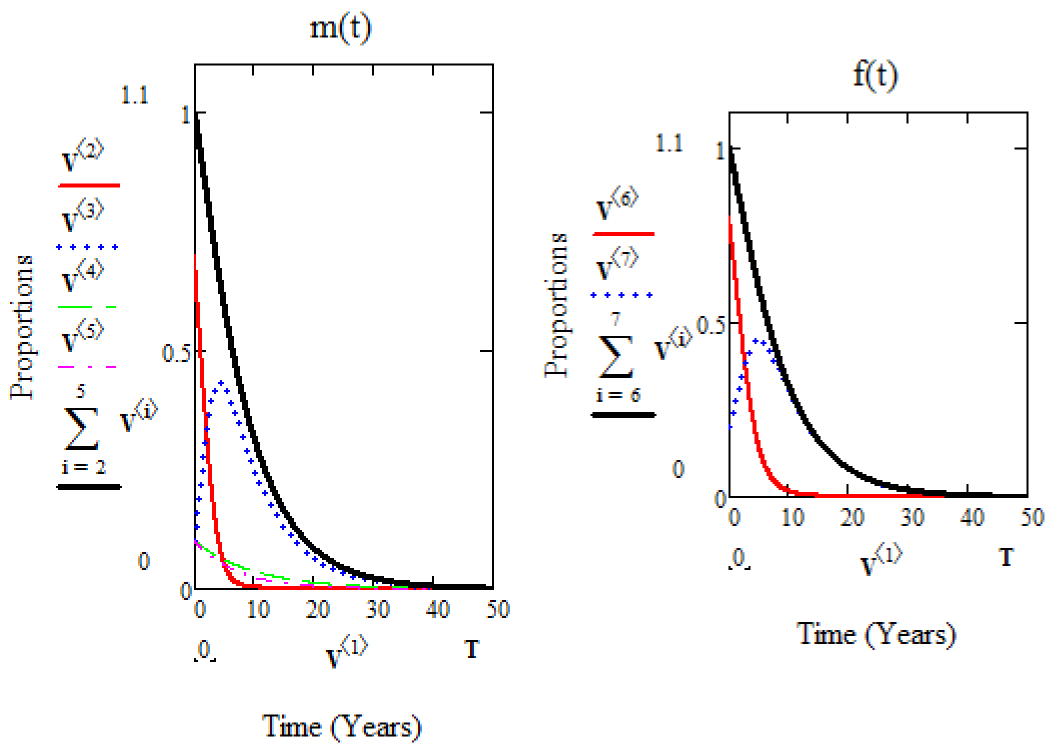

Fig.4.3: Graphical simulations of s_m, y_m, w_m, u_m, s_f and y_f from model (3.11) – (3.16) against time with $V^{\langle i \rangle}$ - the sum of male and female proportions. Parameter values are in variant (3) - Table 3.

Here, if the situation in fig. 4.2, persist with natural death rate of 10% (i.e. $\alpha = 0.1$), as seen in fig 4.3 above, then within a space of 10 years interval, both susceptible male and female population are seen to be consumed by infection which then translate to gradual extinction of the entire population after 30 years' of time interval.

Furthermore, in fig.4.4 below, we investigate the effect of condom use (i.e. $\sigma_1 = 0.2 = \rho_1$), alongside with other induced parameters as in fig. 4.3. We observed a drastic eradication of infection within 18 – 30 years for both the males and female population. The interval taken for complete eradication indicates the low level of condom application and variation of the efficacy of the condom.

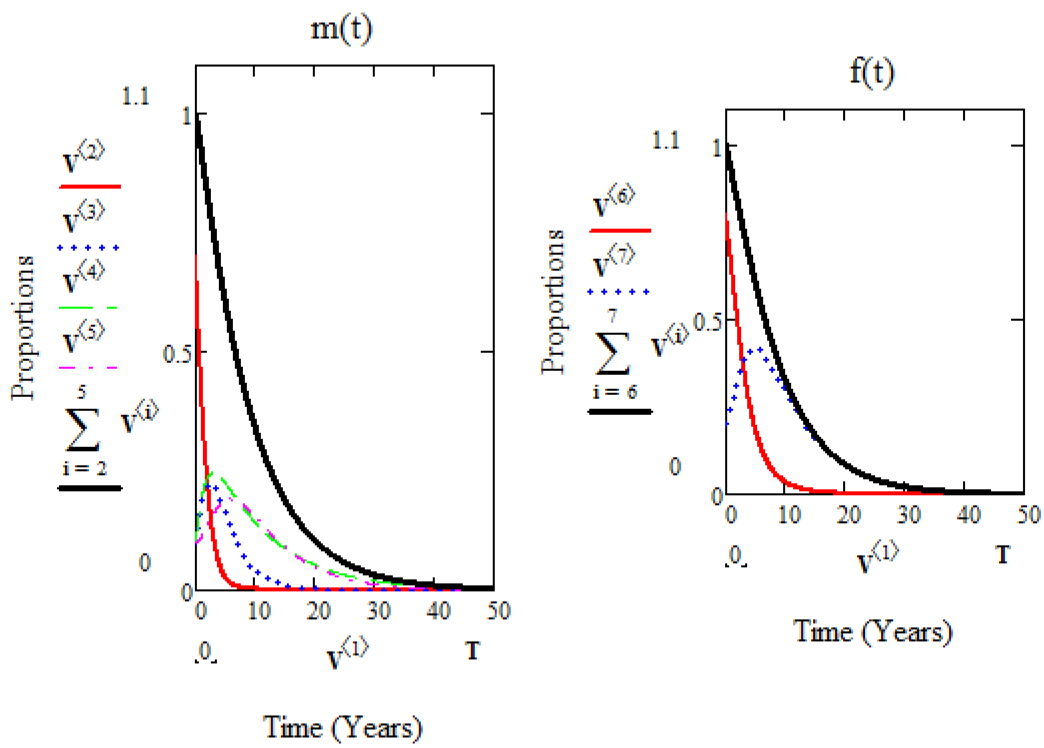

Fig.4.4: Graphical simulations of s_m, y_m, w_m, u_m, s_f and y_f from model (3.11) – (3.16) against time with $V^{\langle i\rangle}$ - the sum of male and female proportions. Parameter values are in variant (4) - Table 3.

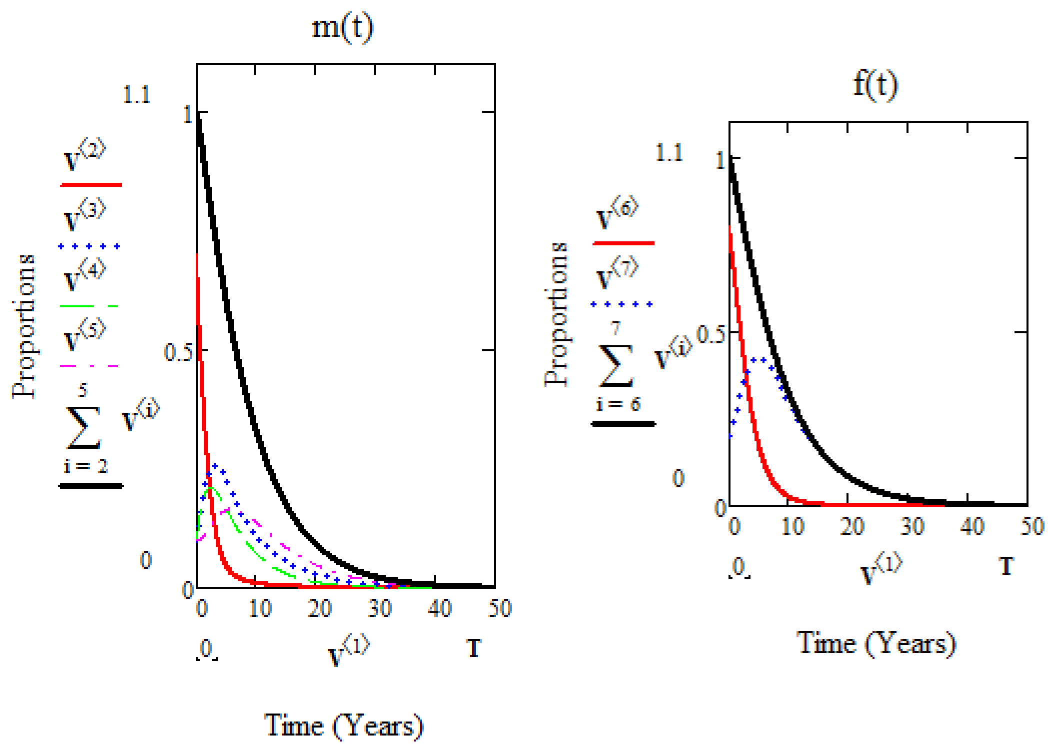

Fig.4.5: Graphical simulations of s_m, y_m, w_m, u_m, s_f and y_f from model (3.11) – (3.16) against time with $V^{\langle i\rangle}$ - the sum of male and female proportions. Parameter values are in variant (5) - Table 3.

Finally, from fig. 4.5 above, we allow the observation as in fig. 4.4, to hold but introduce a non-compliance factor rate of 10% by both males without the condom and those using the condom (i.e. $\sigma_2 = 0.1 = \rho_2$). It is seen that due non-compliance to effective use of the condom, decline of infection from both sexes were slow, taking about 30 years for complete eradication of infection. However, in about ten years on the average, the susceptible had been contaminated by the infection. The situation signified a wholesome non-compliance in condom use and portrays the varying level of condom efficacies.

5. DISCUSSION AND CONCLUSION

In this paper we have extended the model [10] by incorporating in the transmission of HIV, the study of the impact of non-compliance to condom use by both susceptible and infected male population. A non-linear mathematical model was used with the assumption that the population is a two sex and the condom is of varying degree of efficacy. Importantly, in order to appreciate the level of impact of non-compliance of the preventive measure, the model also took into account, natural birth rate b, natural death rate μ and AIDS-related death rate α. We also established the fact that efficacy and non-compliance condom use is dependent on the general behavior of the parameters σ_1, ρ_1 and $\beta^*{}_m$ for efficacy while non-compliance situation are determined by σ_2 and ρ_2.

Using MATHCAD program, results obtained from the numerical simulation of the model equations indicate that eradication of HIV infection cannot not be achieve in a population whereby both the susceptible and infected exhibit a high level of non-compliance in the use of condom as the most effective control program. From the graphical analysis, it is shown that if the rate of compliance is high, the spread of the disease can be reduced significantly. This work is exploratory in nature, for the fact that, it is an improved model that can be used as a correction and conduct of relevant research works, as is in the case of [10] and as well allow the comparison of results.

Furthermore, it is evident from the numerical simulation that, for any effective program, the efficacy and compliance of the condom use should be sufficiently high. Achieving these findings, we recommend intensive and continuous education of the population through all and including media publicity by both government and non-governmental agencies on the effective use of the condom. Any other control measure, ranging from creation of adequate awareness programs, especially in the developing countries and including the study of female condom use, will be highly recommended.

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