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ЭКОНОМИКО-КЛИМАТИЧЕСКАЯ МОДЕЛЬ С ЭНДОГЕННОЙ НОРМОЙ ИЗНОСА КАПИТАЛА В УСЛОВИЯХ НЕОПРЕДЕЛЕННОСТИ ТЕМПЕРАТУРНЫХ ПРОЕКЦИЙ

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Рассмотрена простая экономико-климатическая модель, основанная на *AK*-модели экономического роста с эндогенной нормой износа капитала, линейной по температуре, и с экзогенным климатическим сценарием, при котором температура линейна по времени. Получено аналитическое решение модели. В модели учтена неопределенность температурных проекций, моменты переменной состояния модели рассчитаны аналитически. Изложение проиллюстрировано численными примерами

Ключевые слова: ЭКОНОМИЧЕСКИЙ РОСТ, ИЗМЕНЕНИЕ КЛИМАТА, НЕОПРЕДЕЛЕННОСТЬ

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CLIMATE-ECONOMIC MODEL WITH ENDOGENOUS CAPITAL DEPRECIATION RATE UNDER UNCERTAINTY OF TEMPERATURE PROJECTIONS

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A simple climate–economic model based on the AK model of economic growth with the endogenous depreciation rate linear in temperature and on the exogenous climate scenario with temperature linear in time is considered. The analytical solution of the model is obtained. The uncertainty of temperature projections is introduced in the model, and the moments of model state variable are calculated analytically. Numerical examples are provided

Keywords: ECONOMIC GROWTH, CLIMATE CHANGE, UNCERTAINTY

1. Introduction

It is broadly accepted that climate–economic modelling (Integrated Assessment modelling) inevitably implies substantial uncertainties [8]. One of established approaches for taking into account these uncertainties is performing Monte Carlo simulations with Integrated Assessment models (IAMs) [4, 6, 8]. The idea of the method is that instead of one model run a series of model runs is performed with model parameters that are likely to be responsible for critical uncertainties randomly varying from one model run to another, and then the probability distributions of output model variables of interest are constructed on the basis of these model runs.

Given the complexity of state-of-the-art IAMs, many of which need substantial computational resources, the probability distributions can be obtained only numerically. However, in case of simple ‘toy’ models that have analytical solutions it might be occasionally possible to ‘imitate’ this numeric Monte Carlo procedure by exact analytical calculations of probability distributions and of moments of random output variables of interest. A constructive example of such an ‘imitation’ is provided in the present paper.

The paper is organized as follows. In Sec. 2 a simple climate–economic model based on the AK model with the endogenous depreciation rate and on the exogenous climate scenario is described and its analytical solution is obtained. In Sec. 3 the uncertainty of climate projections is introduced in the model, and the Monte Carlo procedure is imitated. Sec. 4 provides some numerical examples and discussion. Sec. 5 concludes.

2. The AK model with temperature-dependent depreciation rate[1]

The standard AK model is the simplest model of endogenous economic growth [1]. In the AK model the capital $K(t)$ is the state variable, and its dynamics obey an equation

$$\dot{K} = (sA - \delta)K \tag{1}$$

where s is the savings rate, A is the technology parameter and δ is the depreciation rate (all parameters are assumed to be constant). According to Eq. (1), the capital grows exponentially:

$$K(t) = K_0 \exp(r_0 t) \tag{2}$$

where K_0 is the initial capital stock and

$$r_0 = sA - \delta \tag{3}$$

is the (background) growth rate.

[1] This section is based on a part of our previous work: Kovalevsky D.V. Exact analytical solutions of some behaviourist economic growth models with exogenous climate damages. Submitted to: Journal of Mathematical Economics.

Following some previous theoretical work on Integrated Assessment modelling [2, 3, 5, 7], we now assume that the depreciation rate is climate-dependent, and that it increases as global warming evolves. Taking the temperature $T(t)$ as a proxy of the state of the climate system, we assume the linear temperature dependence of the depreciation rate:

$$\delta(t) = \delta_0 \left(1 + \varepsilon (T - T_0)\right) \tag{4}$$

where δ_0 is the initial value of the depreciation rate, ε is the constant sensitivity of depreciation rate to temperature change, and T_0 is the initial value of the temperature. Then Eq. (1) should be rewritten in the form

$$\dot{K} = \left[sA - \delta_0 \left(1 + \varepsilon (T - T_0)\right)\right] K \ , \tag{5}$$

or, equivalently,

$$\dot{K} = \left[r_0 - \delta_0 \varepsilon (T - T_0)\right] K \ , \tag{6}$$

where Eq. (3) has been taken into account.

For the sake of model tractability, we assume a very simple exogenous climate scenario with linear temperature growth:

$$T = T_0 + \Gamma t \ , \qquad \Gamma = \mathrm{const} \ . \tag{7}$$

Then it can be easily shown that the solution of Eq. (6) takes the form

$$K(t) = K_0 \exp\left[r_0 t - \frac{\delta_0 \varepsilon \Gamma}{2} t^2 \right]. \tag{8}$$

3. Impact of uncertainty of temperature projections

We now assume that the parameter Γ is not known with certainty. Instead, we represent it as a sum of its mean value Γ_0 and a random term γ distributed normally with zero mean value and standard deviation σ ($\gamma \propto N(0, \sigma^2)$):

$$\Gamma = \Gamma_0 + \gamma , \tag{9}$$

$$p(\gamma) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{\gamma^2}{2\sigma^2} \right). \tag{10}$$

By substituting the decomposition (9) into Eq. (8) we get

$$K(t) = f(t)g(t,\gamma) \tag{11}$$

where

$$f(t) = K_0 \exp\left[r_0 t - \frac{\delta_0 \varepsilon \Gamma_0}{2} t^2\right], \tag{12}$$

$$g(t,\gamma) = \exp\left[-\frac{\delta_0 \varepsilon t^2}{2}\gamma\right]. \tag{13}$$

It follows immediately from Eq. (11) that the mean value of capital at time t is equal to

$$\overline{K}(t) = f(t)\overline{g}(t) \tag{14}$$

where

$$\overline{g}(t) = \int_{-\infty}^{\infty} g(t,\gamma)p(\gamma)d\gamma\,. \tag{15}$$

By substituting Eqs. (10) and (13) into Eq. (15) and taking the resultant integral we get

$$\overline{g}(t) = \exp\left[\frac{(\delta_0 \varepsilon \sigma)^2}{8} t^4\right]. \tag{16}$$

It follows then from Eqs. (12), (14), and (16) that

$$\overline{K}(t) = K_0 \exp\left[r_0 t - \frac{\delta_0 \varepsilon \Gamma_0}{2} t^2 + \frac{(\delta_0 \varepsilon \sigma)^2}{8} t^4\right]. \tag{17}$$

We now turn to calculation of standard deviation of $K(t)$ ($\sigma_K(t)$). It follows from Eq. (11) that the variance of $K(t)$ is equal to

$$D_K(t) = \sigma_K^2(t) = f^2(t)\sigma_g^2(t) \tag{18}$$

where

$$\sigma_g^2(t) = \overline{g^2}(t) - \overline{g}^2(t) \tag{19}$$

is the variance of $g(t,\gamma)$. The second term in the r.h.s. of Eq. (19) (i.e. $\overline{g}^2(t)$) can be obtained by squaring Eq. (16). Regarding the first term in the r.h.s. of Eq. (19) (i.e. $\overline{g^2}(t)$), we note from Eqs. (13) and (15) that it is provided by the same formula as $\overline{g}(t)$ (i.e. by Eq. (16)) if in the latter the parameter ε is replaced by 2ε:

$$\overline{g^2}(t) = \exp\left[\frac{(\delta_0 \varepsilon \sigma)^2}{2} t^4\right]. \tag{20}$$

Finally,

$$\sigma_g^2(t) = \exp\left[\frac{(\delta_0 \varepsilon \sigma)^2}{4} t^4\right]\left(\exp\left[\frac{(\delta_0 \varepsilon \sigma)^2}{4} t^4\right] - 1\right), \tag{21}$$

and, from Eqs. (12), (17)-(18), and (21)

$$\sigma_K(t) = \overline{K}(t)\sqrt{\exp\left[\frac{(\delta_0 \varepsilon \sigma)^2}{4} t^4\right] - 1}\,. \tag{22}$$

4. Numerical examples and discussion

To provide several numerical results, we adopt the following values of model parameters: $r_0 = 0.02$ year^{-1} (i.e. 2 percent per annum); $\delta_0 = 0.05$ year^{-1} (i.e. 5 percent per annum); $\varepsilon = 0.2$(°C)$^{-1}$ (i.e. the depreciation rate doubles if mean temperature rises by 5°C); $\Gamma_0 = 0.03$°C/year (that corresponds to 3°C warming per century); $\sigma = 0.01$°C/year.

As follows from Eq. (8), the (time-dependent) growth rate

$$r(t) = \frac{\dot{K}(t)}{K(t)} = \frac{d}{dt}\ln K(t) \tag{23}$$

in the case of no uncertainty is equal to

$$r(t) = r_0 - \delta_0 \varepsilon \Gamma_0 t\,, \tag{24}$$

so it decreases linearly in time and eventually becomes negative. At the same time, according to Eq. (17), the time-dependent growth rate of mean value of capital in the case of uncertainty

$$r_\sigma(t) == \frac{d}{dt}\ln \overline{K}(t) \tag{25}$$

is equal to

$$r_\sigma(t) = r_0 - \delta_0 \varepsilon \Gamma_0 t + \frac{(\delta_0 \varepsilon \sigma)^2}{2} t^3, \tag{26}$$

and it increases at large times. The graphs of $r(t)$ and $r_\sigma(t)$ are shown on Fig. 1.

According to Eq. (8), in case of no uncertainty the time dependence of capital $K(t)$ itself is given by a Gaussian curve: it first increases, reaches its maximum at $t = T^*/2$ and then starts decreasing, dropping to the initial value K_0 at $t = T^*$ and ultimately converging to zero at infinite time. Here

$$T^* = \frac{2r_0}{\delta_0 \varepsilon \Gamma_0}, \tag{27}$$

and for numerical values of parameters specified above T^* is equal to 133 years. The graph of normalized capital $K(t)/K_0$ is shown on Fig. 2.

Finally, the graph of normalized mean value of capital $\overline{K}(t)/K_0$ in case of uncertainty is shown on Fig. 3. As seen on the figure, it rapidly increases at large times.

So why the solution in case of no uncertainty is so different in the long run from the average solution in case of uncertainty? The reason is in that the values of a random variable γ from a 'symmetric' couple ($-\gamma$, γ) contribute asymmetrically ('unevenly') to the resultant moments: lower-than-average temperature growth rates contribute more than higher-than-average ones. This means that the resultant average solutions are shifted from high-end scenarios to low-end ones when climate change matters less, and therefore the average solution in case of uncertainty is overly much more optimistic in the long run that the solution in case of no uncertainty.

5. Conclusions

A simple climate–economic model described in Sec. 2 has the exact analytical solution. Moreover, its uniqueness is that it allows introducing the uncertainty in a tractable way and calculating probability distributions and moments of model state variable in closed analytical form as well. As the discussion in Sec. 4 has shown, the model yields instructive results. We believe performing the same procedure with other 'toy' IAMs that can be solved analytically and semi-analytically would be an interesting direction of further research. One important issue that should be addressed in such modelling exercises is going beyond symmetric distributions for model parameters (like normal distribution adopted in the present paper) to address the problem of ‘fat tails’ that is currently topical in economics of climate change.

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Литература

1. Барро Р.Дж., Сала-и-Мартин Х. Экономический рост; пер. с англ. М.: БИНОМ, Лаборатория знаний, 2010. 824 с.
2. Ковалевский Д.В. Экономико-климатическая модель с растущей нормой амортизации производственных фондов // Научно-технические ведомости СПбГПУ. Серия: Экономические науки. 2011. № 6(137). С. 218-221.
3. Ковалевский Д.В., Бобылев Л.П. Эффект эндогенности национальных норм амортизации производственных фондов в регионализованной экономико-климатической модели // Научно-технические ведомости СПбГПУ. Серия: Экономические науки. 2012. № 5(156). С. 176-179.
4. Ackerman F., Stanton E.A., Bueno R. Fat tails, exponents, extreme uncertainty: Simulating catastrophe in DICE // Ecological Economics. 2010. Vol. 69. P. 1657–1665.
5. Bretschger L., Valente S. Climate change and uneven development // The Scandinavian Journal of Economics. 2011. Vol. 113. P. 825-845.
6. Dietz S. High impact, low probability? An empirical analysis of risk in the economics of climate change // Climatic Change. 2011. Vol. 108. P. 519–541.
7. Ikefuji M., Horii R. Natural disasters in a two-sector model of endogenous growth // Journal of Public Economics. 2012. Vol. 96. P. 784-796.
8. Stern N. The Stern Review on the Economics of Climate Change. Cambridge University Press, Cambridge, 2006.

References

1. Barro R.Dzh., Sala-i-Martin H. Jekonomicheskij rost; per. s angl. M.: BINOM, Laboratorija znanij, 2010. 824 s.
2. Kovalevskij D.V. Jekonomiko-klimaticheskaja model' s rastushhej normoj amortizacii proizvodstvennyh fondov // Nauchno-tehnicheskie vedomosti SPbGPU. Serija: Jekonomicheskie nauki. 2011. № 6(137). S. 218-221.
3. Kovalevskij D.V., Bobylev L.P. Jeffekt jendogennosti nacional'nyh norm amortizacii proizvodstvennyh fondov v regionalizovannoj jekonomiko-klimaticheskoj modeli

// Nauchno-tehnicheskie vedomosti SPbGPU. Serija: Jekonomicheskie nauki. 2012. № 5(156). S. 176-179.

4. Ackerman F., Stanton E.A., Bueno R. Fat tails, exponents, extreme uncertainty: Simulating catastrophe in DICE // Ecological Economics. 2010. Vol. 69. P. 1657–1665.

5. Bretschger L., Valente S. Climate change and uneven development // The Scandinavian Journal of Economics. 2011. Vol. 113. P. 825-845.

6. Dietz S. High impact, low probability? An empirical analysis of risk in the economics of climate change // Climatic Change. 2011. Vol. 108. P. 519–541.

7. Ikefuji M., Horii R. Natural disasters in a two-sector model of endogenous growth // Journal of Public Economics. 2012. Vol. 96. P. 784-796.

8. Stern N. The Stern Review on the Economics of Climate Change. Cambridge

FIGURES

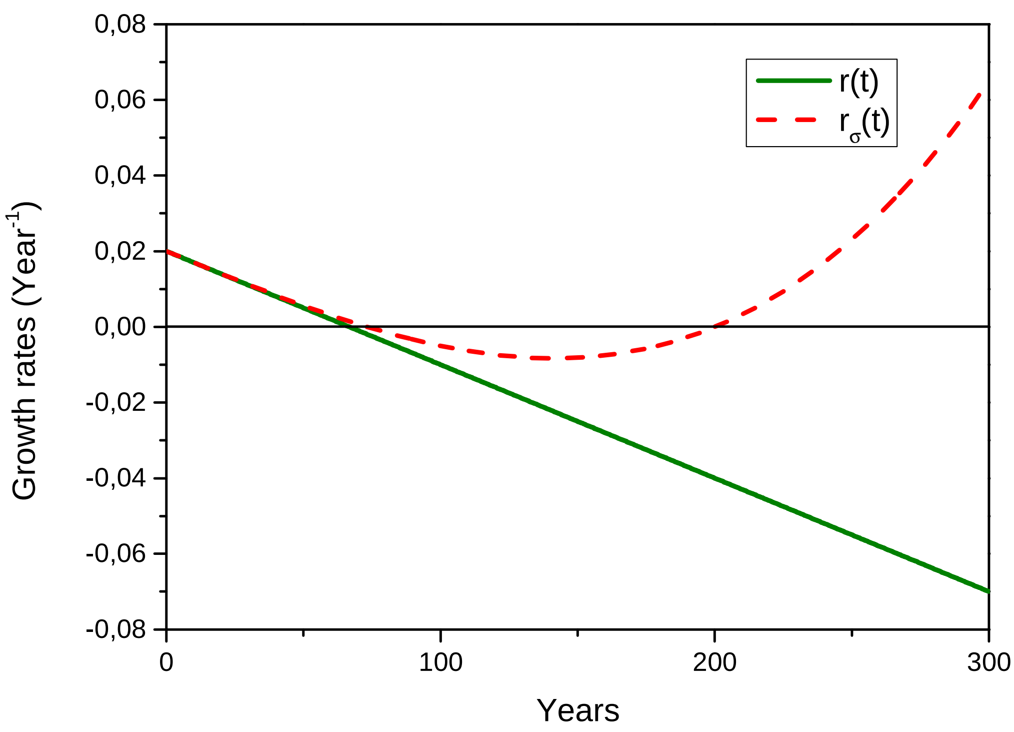

Figure 1: The time-dependent growth rates in case with no uncertainty ($r(t)$, solid green line) and with uncertainty ($r_\sigma(t)$, dashed red line).

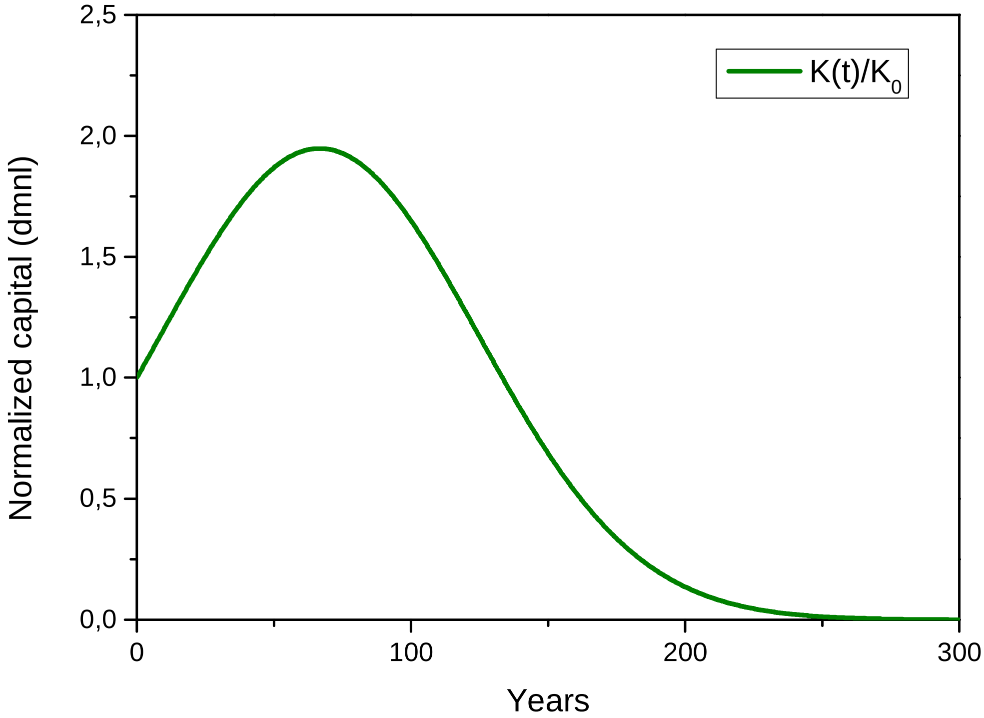

Figure 2: Normalized capital dynamics in case of no uncertainty.

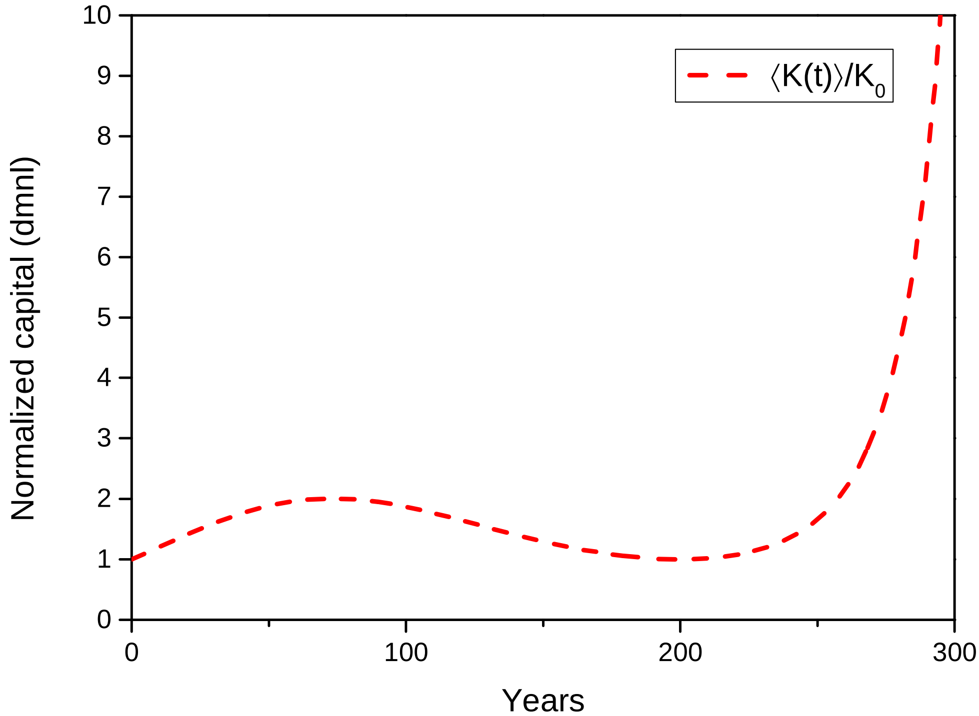

Figure 3: The dynamics of mean value of normalized capital in case of uncertainty.