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ЯДЕРНЫЕ ОБОЛОЧКИ И ПЕРИОДИЧЕСКИЙ ЗАКОН Д.И.МЕНДЕЛЕЕВА

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На основе теории ядерных взаимодействий и данных по энергии связи нуклонов для всех известных нуклидов установлены параметры, характеризующие периодические закономерности в формировании ядерных оболочек

Ключевые слова: НЕЙТРОН, ПЕРИОДИЧЕСКИЙ ЗАКОН, ПРОТОН, ЭНЕРГИЯ СВЯЗИ, ЯДРО

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NUCLEI SHELLS AND PERIODIC TRENDS

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Parameters describing periodic trends in the formation of nuclear shells have been established based on the theory of nuclear interactions and data on the binding energy of nucleons for the set of known nuclides

Keywords: BINDING ENERGY, PERIODIC TRENDS, PROTON, NEUTRON, NUCLEI

Introduction

The periodic law discovered by Mendeleev in 1869, played a huge role in the development of ideas about the structure of matter. In one of the first formulations of this law states that "the properties of simple bodies, as well as the shape and properties of the compounds of the elements, and therefore the properties of which they form simple and complex bodies are in the periodic table according to their atomic weight" [1]. Using this law, Mendeleev created the periodic table of elements, but also predicted new elements (the modern name of these elements - gallium, scandium, germanium, and astatine), which were later discovered.

Nowadays initial formulation of the periodic law has undergone significant change, in accordance with a change in ideas about the structure of atoms of chemical elements. In the modern formulation proposed by Antonius Van den Broek [2] in 1911, this law states that "the properties of simple substances, as well as the shape and properties of the compounds of the elements are in periodic dependence on the charges of the nuclei of atoms of elements." Note that this formulation is directly related to the model of the atom, which arose due to the experimental discovery of Rutherford [3] and his student Moseley [4]. In this model, the atom consists of a central, positively charged nucleus and electron shells. The rule of electron shells filled, which is formulated by Pauli in 1925, became the basis for new interpretations of the periodic law. In the modern periodic table of chemical elements made enter, along with the atomic weight and atomic number, the configuration of the ground state and the ionization energy - Table 1.

Table 1: A fragment of the periodic table of chemical elements according to [5]

Z	Element	The configuration of the ground-state					Ground state	Ionization energy, eV
78	Pt	[Xe]	$4f^{14}$	$5d^9$	$6s$		$^3D_3$	8,9588
79	Au	[Xe]	$4f^{14}$	$5d^{10}$	$6s$		$^2S_{1/2}$	9,2255
80	Hg	[Xe]	$4f^{14}$	$5d^{10}$	$6s^2$		$^1S_0$	10,4375
81	Tl	[Xe]	$4f^{14}$	$5d^{10}$	$6s^2$	$6p$	$^2P^o_{1/2}$	6,1082
82	Pb	[Xe]	$4f^{14}$	$5d^{10}$	$6s^2$	$6p^2$	$^3P_0$	7,4167
83	Bi	[Xe]	$4f^{14}$	$5d^{10}$	$6s^2$	$6p^3$	$^4S^o_{3/2}$	7,2855
84	Po	[Xe]	$4f^{14}$	$5d^{10}$	$6s^2$	$6p^4$	$^3P_2$	8,414
85	At	[Xe]	$4f^{14}$	$5d^{10}$	$6s^2$	$6p^5$	$^2P^o_{3/2}$	9,5
86	Rn	[Xe]	$4f^{14}$	$5d^{10}$	$6s^2$	$6p^6$	$^1S_0$	10,7485
87	Fr	[Rn]			$7s$		$^2S_{1/2}$	4,0727
88	Ra	[Rn]			$7s^2$		$^1S_0$	5,2784
89	Ac	[Rn]		$6d$	$7s^2$		$^2D_{3/2}$	5,3807
90	Th	[Rn]		$6d^2$	$7s^2$		$^3F_2$	6,3067

At first glance, the two formulations of the periodic law equivalent, in any case, if we assume that the atomic weight and charge of the nucleus are linked monotonic dependence. However, the presence of isotopes clearly violates this equivalence, as the core of isotopes of one element are different numbers of neutrons, with an equal number of protons, i.e. isotopes have different atomic weight of an equal charge of the nucleus. Consequently, the masses of isotopes of one element differ from each other, that should have been, because of the periodic law of Mendeleev's original interpretation, affect their chemical properties. But this is in contradiction with the quantum theory, in which it is assumed that the structure of electron shells depends on the nuclear charge only.

The question arises, what exactly the property of atomic nuclei is manifested in the periodic law? This question can be formulated in very general terms. For this we consider the atomic nucleus, consisting, according to the hypothesis Ivanenko [6] and Heisenberg [7], of N neutrons and Z protons. The total number of nucleons is denoted $A = Z + N$. This nucleus has an electric charge eZ, and its mass is expressed as $M_A = m_p Z + m_n N - E_b / c^2$, where - E_b is the binding energy of the nucleons in the nucleus. Assume that the properties of matter, one of which is denoted P_{AZ} , are defined according to the law of Mendeleev. In this case we have the dependence

$$P_{AZ} = P_{AZ}(M_A) \tag{1}$$

Note that the dependence (1) is empirical one. It was obtained by many experiments in chemistry for a century. If you use a modern formulation of the periodic law, then the property will also describe the dependence of the type

$$P_{AZ} = P_{AZ}(Z) \tag{2}$$

Dependence (2) is also empirical. It is based on well-known law Moseley [4] for the X-ray spectra of atoms and numerous other types of periodic trends depending on the ionization energy of the atomic number - Fig. 1. In theory, these periodic patterns are explained, mainly, the rule of filling of electron shells, which is derived from Schrödinger quantum mechanics, and Pauli principle.

Ivanenko [8], apparently, was one of the first who raised the issue of expansion of the periodic law to include the periodic patterns are observed in atomic nuclei and some exotic formations, such as exotic atoms. He started from the theory of atomic shells [7, 9], which at that time was experiencing a period of dawn. According to this theory, the periodic patterns in the nuclei are explained by analogy with the electron shells, the Pauli principle, which is applied separately for protons and neutrons fill the nuclear shells. It is clear that with this expansion of the periodic law of its original formulation (1) seems more logical, since the properties of the nuclei depends not only on the number of protons but also on the number of neutrons. However, the expression (2) also has a well-defined area of application in the interpretation of the periodic properties of chemical elements. The main thing is that the properties of nuclei, and properties of atoms of chemical elements due to the same type on the basis of quantum mechanics and the Pauli principle [8].

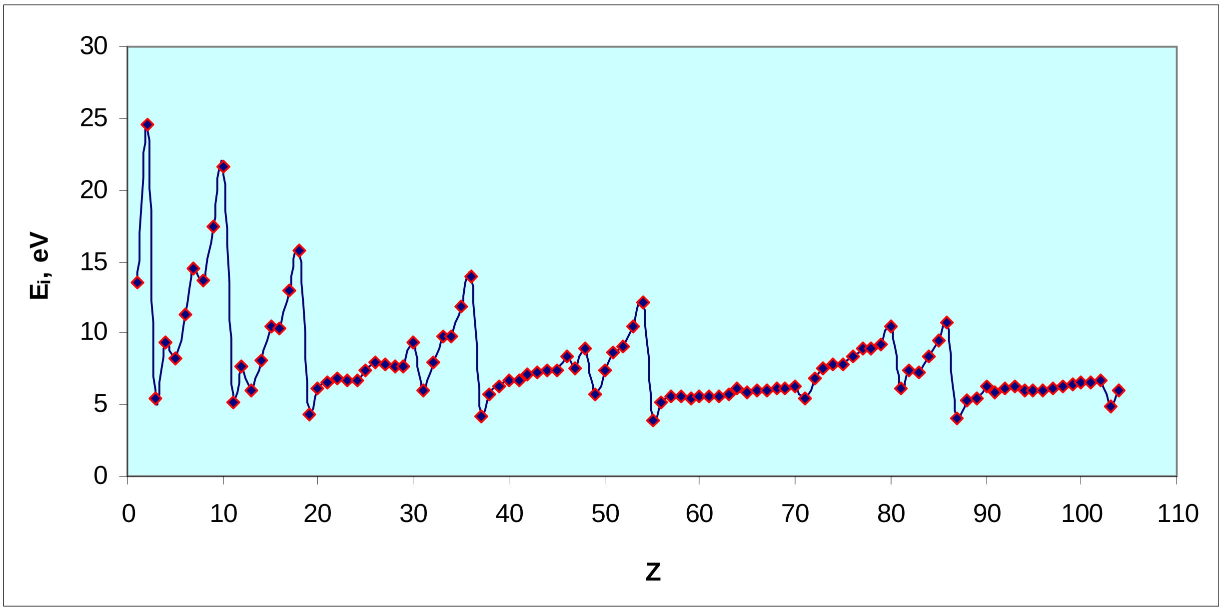

Fig. 1: The dependence of the ionization energy of the atomic number of the data [5].

The reduction of the Mendeleev's periodic law to the Pauli principle and quantum mechanics does not solve the problem itself the presence of periodic patterns, but only reduces it to a hypothetical description of the problem of atomic and nuclear shells. But now the problem of quantum-mechanical description of atomic shells of many-electron atoms is far from being solved. For example, there is no quantum-mechanical model to calculate satisfactory the ionization energy of many-electron atoms, shown in Fig. 1. On the contrary, the theory of Mills [10], based on classical electrodynamics and relativistic mechanics, allows us to calculate the binding energy of many-electron atoms up to 20 electrons with very high accuracy - Table 2.

Table 2: The ionization energy of some 19 and 20-electron atoms calculated on the model [10].

19 e Atom	Z	Theoretical Ionization Energies (eV)	Experimental Ionization Energies (eV)	Relative Error	20 e Atom	Z	Theoretical Ionization Energies (eV)	Experimental Ionization Energies (eV)	Relative Error
K	19	4.32596	4.34066	0.0034					
Ca^{+}	20	11.3354	11.87172	0.0452	Ca	20	6.10101	6.11316	0.0020
Sc^{2+}	21	24.6988	24.75666	0.0023	Sc^{+}	21	13.2824	12.79967	-0.0377
Ti^{3+}	22	41.8647	43.2672	0.0324	Ti^{2+}	22	27.4719	27.4917	0.0007
V^{4+}	23	62.8474	65.2817	0.0373	V^{3+}	23	45.6956	46.709	0.0217
Cr^{5+}	24	87.6329	90.6349	0.0331	Cr^{4+}	24	67.8794	69.46	0.0228
Mn^{6+}	25	116.2076	119.203	0.0251	Mn^{5+}	25	93.9766	95.6	0.0170
Fe^{7+}	26	148.5612	151.06	0.0165	Fe^{6+}	26	123.9571	124.98	0.0082
Co^{8+}	27	184.6863	186.13	0.0078	Co^{7+}	27	157.8012	157.8	0.0000
Ni^{9+}	28	224.5772	224.6	0.0001	Ni^{8+}	28	195.4954	193	-0.0129
Cu^{10+}	29	268.2300	265.3	-0.0110	Cu^{9+}	29	237.0301	232	-0.0217
Zn^{11+}	30	315.6418	310.8	-0.0156	Zn^{10+}	30	282.3982	274	-0.0307

It was shown that the Mills theory [10] can be derived from Lorentz quantum electrodynamics [11], obtained by combining the Lorentz classical electrodynamics and the Klein-Gordon equation. Consequently, the Mills theory does not contradict quantum mechanics, especially since it uses the same Pauli principle and hypothesis of the existence of electronic shells, which are assumed to be infinitely thin in the radial direction. The radii of the shells are calculated on the basis of the classical equations of motion of the electron, taking into account the quantum effects of spin and angular momentum (here seen an obvious connection with Bohr's theory). Thus, the results obtained on the basis of the Mills theory, confirm the hypothesis van den Broek of the dependence of the properties of ordinary matter on the nuclear charge in the form (2).

We may notice that the periodic law in the original formulation of Mendeleev is local, as relates properties of simple substances with their atomic weight, which at the time when the law was formulated, was determined by weighing in the gravitational field of the Earth. Such a correlation properties of substances and their gravitational properties seem reasonable because in macroscopic chemical experiments, the nuclear charges are not observed. In this sense, the correlation properties of simple substances with the charge of their nuclei in the form (2) is somewhat abstract, even though this hypothesis and allows you to arrange the chemical elements according to the configuration of the atomic shells - Table 1. But as shown above, the electron shell of itself is more a theoretical than a real phenomenon, and even its theoretical description is not unique, as evidenced by Mills theory [10].

In addition, the wording of the periodic law in the form of (1) or (2) involves only the properties of the nuclei of elements, but not the properties of electrons filling the electronic shells. Apparently, we cannot argue that the properties of electrons in any way affect the properties of simple substances; otherwise, these properties must be reflected in the wording of the periodic law.

In the present work is given extended treatment of the periodic law, combining both his statements into one that allows you to organize all the chemical elements and their isotopes in accordance with a set of quantum numbers characterizing the nuclear shells. For this purpose we developed the model of nuclear interactions [12], based on the five-dimensional unified theory of gravitation and electromagnetism [13-14] and the theory of fundamental interactions [15-16].

Description of the model

To answer the question about the fundamental causes that lead to a law of periodicity in nature, consider a general model of atomic nuclei and atoms of matter [12]. In this model, the properties of matter are determined by the parameters of the metric tensor in 5-dimensional space, which depend on a combination of charge and gravitational properties of the central core in the form

$$k = 2gM_A^3 c^2 / Q^4, \quad e^2 / k = 2gM_A / c^2 \tag{3}$$

Here γ, c, Q are the gravitational constant, the speed of light and charge of the nucleus, respectively. About the nature of the charge will be assumed that the source is an electric charge, but it can be screened in various natural fields. The mechanism of screening and related fields is discussed below. In the case of proton and electron parameters of the metric tensor (3) are presented in Table 3.

Table 3: Parameters of the metric tensor

	k, 1/m	e	r_{max}, m	r_{min}, m
e-	1,703163E-28	4,799488E-43	5,87E+27	2,81799E-15
p+	1,054395E-18	1,618178E-36	9,48E+17	1,5347E-18

Note that the maximum scale $r_{\max} = 1/k$ in the case of an electron exceeds the size of the observable universe, while this scale is about 100 light-years for protons. The minimum is the scale $r_{\min} = e/k = e^2/mc^2$ corresponds to the classical radius of a charged particle, which in the case of the proton and the electron is comparable to the scale of weak and nuclear interactions.

It is easy to see that the second parameter of the model (3) enters directly into the formula of the periodic law in the form of (1). Combining the parameters, we find the nuclear charge $Q = e^{3/2} c^2 / k\sqrt{2g}$. Consequently, the periodic law of the form (2) can also be expressed through the parameters of the metric tensor (3).

The metric tensor can be expanded in the vicinity of a massive center of gravity in five-dimensional space in powers of the distance from the source $r = \sqrt{x^2 + y^2 + z^2}$, in the form

$$G_{ik} = G_{ik}(0) + \dot{G}_{ik}(0)\tilde{r} + \ddot{G}_{ik}(0)\frac{\tilde{r}^2}{2} + ... \quad (4)$$

Here the dot denotes differentiation with respect to the dimensionless parameter $\tilde{r} = kr$. Consider the form of the tensor (4) that occurs when holding the first three terms in the expansion of the metric in the case of central force field with the gravitational potential in the form of Newton. This choice of metric is justified, primarily because of the specified building the superposition principle holds. Suppose $x^1 = ct, x^2 = x, x^3 = y, x^4 = z$, in this notation we have for the square of the interval in the 4-dimensional space:

$$\begin{aligned} & ds^2 = (1 + 2j / c^2)c^2 dt^2 - (1 - 2j / c^2)(dx^2 + dy^2 + dz^2) \\ & j = -\frac{gM}{r} \end{aligned} \quad (5)$$

Assume that coefficients of the metric in the five-dimensional space are characterized by a parameter $e^2 = G_{11}(0) = -\dot{G}_{11}(0)$ (this assumption is equivalent to the classical relation between the mass and charge of an electron). Then, assuming that $e^2 / k = 2gM / c^2$, we get the expression of the interval depending on the parameters of the metric in the five-dimensional space:

$$ds^2 = (1 - e^2 / k)c^2 dt^2 - (1 + e^2 / k)(dx^2 + dy^2 + dz^2) \quad (6)$$

Further, we note that in this case the metric tensor in four dimensions is diagonal with components

$$g_{11} = 1 - e^2 / kr; \quad g_{22} = g_{33} = g_{44} = -(1 + e^2 / kr) \quad (7)$$

We define the vector potential of the source associated with the center of gravity in the form

$$g_1 = e / kr, \quad \mathbf{g} = g_1 \mathbf{u} \quad (8)$$

Here $\mathbf{u}$ is a vector in three dimensional spaces, which we define below. Hence, we find the scalar and vector potential of electromagnetic field

$$j_e = \frac{Q}{r} = \frac{Mc^2}{e}\frac{e}{kr}, \quad \mathbf{A} = j_{\mathbf{e}} \mathbf{u} \quad (9)$$

Setting $h=(kr)^2$ and evaluating the metric tensor in 5-dimensional space, using (7) - (8), we find that in this case, expression (4) contains in the right side, only three terms of the series expansion in powers of $\tilde{r}=kr$, thus

$$G_{ik}=h\begin{pmatrix} g_{ik}+g_i g_k & g_k \\ g_i & 1 \end{pmatrix}=G_{ik}(0)+\dot{G}_{ik}(0)\tilde{r}+\ddot{G}_{ik}(0)\frac{\tilde{r}^2}{2} \tag{10}$$

To describe the motion of matter in the light of its wave properties, we assume that the standard Hamilton-Jacobi equation in the relativistic mechanics and Klein-Gordon equation in quantum mechanics arise as a consequence of the wave equation in five-dimensional space [14, 16]. This equation can generally be written as:

$$\frac{1}{\sqrt{-G}}\frac{\partial}{\partial x^m}\left(\sqrt{-G}G^{mn}\frac{\partial}{\partial x^n}\Psi\right)=0 \tag{11}$$

Here Ψ is the wave function describing, according to (11), the scalar field in five-dimensional space, and G^{ik} is the contravariant metric tensor,

$$G^{ik}=h^{-1}\begin{pmatrix} l_1 & 0 & 0 & 0 & -g^1 \\ 0 & l_2 & 0 & 0 & -g^2 \\ 0 & 0 & l_2 & 0 & -g^3 \\ 0 & 0 & 0 & l_2 & -g^4 \\ -g^1 & -g^2 & -g^3 & -g^4 & l \end{pmatrix} \tag{12}$$

$$l_1=(1-e^2/kr)^{-1};\quad l_2=-(1+e^2/kr)^{-1}$$
$$g^1=l_1 g_1,\ g^2=l_2 g_2,\ g^3=l_2 g_3,\ g^4=l_2 g_4$$
$$l=1+l_1 g_1^2+l_2(g_2^2+g_3^2+g_4^2);\quad G=h^5/(ab^3);\quad h=(kr)^2.$$

We further note that in the investigated metrics, depending only on the radial coordinate, the following relation is true

$$F^m=h\frac{\partial}{\partial x^m}\left(\sqrt{-G}G^{mn}\right)=h\frac{\partial r}{\partial x^m}\frac{d}{dr}\left(\sqrt{-G}G^{mn}\right) \tag{13}$$

Taking into account expressions (12), (13), we write the wave equation (11) as

$$\frac{l_1}{c^2}\frac{\partial^2\Psi}{\partial t^2}-|l_2|\nabla^2\Psi+\lambda\frac{\partial^2\Psi}{\partial\rho^2}-2g^i\frac{\partial^2\Psi}{\partial x^i\partial\rho}+F^\mu\frac{\partial\Psi}{\partial x^\mu}=0 \tag{14}$$

Note that the last term in equation (14) is of the order $h^2 k = k^5 r^4 << 1$. Consequently, this term can be dropped in the problems, the characteristic scale which is considerably less than the maximum scale in Table 1. Equation (14) is remarkable in that it does not contain any parameters that characterize the scalar field. The field acquires a mass and charge, not only electric, but also strong in the process of interaction with the central body, which is due only to the metric of 5-dimensional space [12, 16].

Consider the problem of the motion of matter around the charged center of gravity, which has an electrical and strong charge, for example, around the proton. In the process of solving this problem is necessary to define the inertial mass of matter and binding energy. Since equation (14) is linear and homogeneous, this problem can be solved in general.

We introduce a polar coordinate system (r, f, z) with the z axis is directed along the vector potential (8), we put in equation (14)

$$\Psi = y\,(r)\exp(ilf + ik_z z - iwt - ik_r\, r) \tag{15}$$

Separating the variables, we find that the radial distribution of matter is described by the following equation (here we dropped, because of its smallness, the last term in equation (14)):

$$-\frac{l_1 w^2}{c^2} y - |\,l_2\,| \left(y_{\,rr} + \frac{1}{r} y_{\,r} - \frac{l^2}{r^2} y - k_z^2 y \right) - \lambda k_r^2 y + 2\mathrm{g}^1 c^{-1} w k_r y - 2 g^z k_z k_r y = 0$$

(16)

Consider the solutions (16) in the case when one can neglect the influence of gravity, i.e. $l_1 \approx -l_2 \approx 1$, but $l = 1 + g_1^2 (1 - u^2) \neq 1$. Under these conditions, equation (16) reduces to

$$-\frac{w^2}{c^2} y - \left(y_{\,rr} + \frac{1}{r} y_{\,r} - \frac{l^2}{r^2} y - k_z^2 y \right) - \lambda k_r^2 y + 2\mathrm{g}^1 c^{-1} w k_r y - 2 g^z k_z k_r y = 0$$

(17)

In general, the solution of equation (17) can be represented in the form of power series, as in the analogous problem of excited states of the relativistic hydrogen atom [17-18]

$$y = \frac{\exp(-\tilde{r})}{\tilde{r}^a}\sum_{j=0}^{n} c_j \tilde{r}^j \tag{18}$$

It is indicated $\tilde{r} = r / r_n$. Substituting (18) in equation (17), we find

$$(a^2 - l^2 + k_u)\sum_{j=0}^{n} c_j \tilde{r}^{j-2} + (2a - 1 + k_g r_n)\sum_{j=0}^{n} c_j \tilde{r}^{j-1} +$$
$$(1 - k_z^2 r_n^2 + K^2 r_n^2)\sum_{j=0}^{n} c_j \tilde{r}^j - \sum_{j=0}^{n} j c_j \tilde{r}^{j-1} - 2a\sum_{j=0}^{n} j c_j \tilde{r}^{j-2} +$$
$$\sum_{j=0}^{n} c_j j(j-1)\tilde{r}^{j-2} = 0 \tag{19}$$

$k_u = (1 - u^2)k_r^2 e^2 / k^2$, $K^2 = k_r^2 + w^2 / c^2$, $k_g = -2ek_r(k_z u_z + w/c)/k > 0$.

Hence, equating coefficients of like powers $\tilde{r} = r / r_n$, we obtain the equations relating the parameters of the model in the case of excited states

$$a = \sqrt{l^2 - k_u}, \quad r_n = \frac{n + 1 - 2a}{k_g}, \quad \frac{1}{r_n^2} - k_z^2 + k_r^2 + \frac{w^2}{c^2} = 0 \tag{20}$$

The second equation (20) holds only for values of the exponent, for which the inequality $2a < n + 1$ is true. Hence, we find an equation for determining the energy levels

$$\frac{4e^2 k_r^2}{k^2(n + 1 - 2a)^2}\left(k_z u_z + \frac{w}{c}\right)^2 - k_z^2 + k_r^2 + \frac{w^2}{c^2} = 0 \tag{21}$$

Equation (21) was used to model the binding energy of nucleons in the nucleus for the entire set of known nuclides [12]. In the model [12], the nucleus consists of protons, interacting with a scalar field. Part of the proton is screened by forming N neutrons, as a result there is an atom, consisting of the electron shell and nucleus with electric charge eZ, number of nucleons $A = Z + N$ and the mass $M_A = A(m_p + m_e) - E_{bp} / c^2$, where E_{bp} is the energy of the nucleons in the nucleus, which is calculated on the total number of nucleons, with a total mass of the electron and proton. Note that this expression of binding energy is not essential one, so we can use standard expression of the mass excess in atomic units. Since in this problem two types of charges appear - scalar and vector, the effect of screening manifests itself not only with respect to the scalar charge (which leads to the formation of neutrons), but also in terms of the vector charge, which leads to the formation of the nucleons.

It should be noted that the original metric in the five-dimensional space defined by the metric tensor, which depends only on the parameters of the central body, ie of the total charge and total mass of the nucleons. Depending on the combination of the charge and mass of the nucleus in different shells can be formed:

1) Nucleon shell, in which all charges are screened, therefore - $e / k = A^2 e^2 / A m_p c^2 = A e^2 / m_p c^2$;

2) Neutron shell - $e / k = N e^2 / m_p c^2$;

3) Proton shell - $e / k = Z e^2 / m_p c^2$.

Using the electron mass and Planck's constant, we define the dimensionless parameters of the model in the form

$$a = \frac{e^2}{\mathbf{h}c}, \; S = \frac{(\mathbf{h}k_r)^2}{(m_e c)^2}, \; P = \frac{\mathbf{h}k_z}{m_e c}, \; E = \frac{\mathbf{h}w}{m_e c^2}$$

$$b_{nl}^X = \frac{4X^2 (a m_e / m_p)^2}{\left(n + 1 - 2\sqrt{l^2 - (1 - u^2) S X^2 (a m_e / m_p)^2}\right)^2} \tag{22}$$

Here $X = A, N, Z$, in the case of the nucleon, neutron and proton shells, respectively.

Solving equation (21) with respect to energy, we find

$$E_{nl}^{X} = \frac{-Sb_{nl}^{X}Pu \pm i\sqrt{-(Sb_{nl}^{X}Pu)^{2} + (Sb_{nl}^{X}+1)(S - P^{2} + Sb_{nl}^{X}P^{2}u^{2})}}{(Sb_{nl}^{X}+1)} \tag{23}$$

Note that the parameter in the energy equation (23) can be both real and complex values, which correspond to states with finite lifetime. Given that for most nuclides the decay time is large enough quantity, so it can be assumed that the imaginary part of the right-hand side of equation (23) is a small value, which corresponds to a small value of the radicand. Hence we find that for these states the following relation between the parameters

$$P^{2} = \frac{S(Sb_{nl}^{X}+1)}{1 + Sb_{nl}^{X}(1-u^{2})} \tag{24}$$

Substituting (24) in equation (23), we have

$$E_{nl}^{X} = \frac{S^{3/2}b_{nl}^{X}u}{\sqrt{(Sb_{nl}^{X}+1)(1+Sb_{nl}^{X}(1-u^{2}))}} \tag{25}$$

Hence, we find the dependence of the binding energy per nucleon in the ground state

$$E_{0a}^{X}/A = \frac{S^{3/2}b_{0}uX^{2}/A}{\sqrt{(Sb_{0}X^{2}+1)(1+Sb_{0}X^{2}(1-u^{2}))}} \tag{26}$$

It is indicated $b_0 = (2a\, m_e / m_p (1-2a))^2$. Thus, we have established a link between the energy of the state and parameters of the interaction. Note that the binding energy (25) depends on the magnitude of the vector charge, which appears in equations (8) - (9). As shown below, this demonstrates the difference between the interaction of nucleons in nuclei, where the vector charge parameter $u \neq 0$, and the interaction between electrons and atomic nuclei, in which $u = 0$.

Nuclear shells and the binding energy of nucleons

For the best agreement with the data model [19] in the case of the nucleon shell, put in the equation (26): $X = A$, $\sqrt{S} = 293$; $Sb_0 = 0.003$; $u = 0.9986$. Equation (26) can approximately describe the dependence of the binding energy of the number of nucleons for all nuclides – Figure 2. For light nuclei, there is a significant discrepancy with experiment. This is due both to the presence of two other shells, and the fact that the structure of light nuclei is largely dependent on the details of the interaction. In particular, parameters S, b_0, u are not constant, as shown below.

In this regard, we note that in theory [14], the action in the five-dimensional space can be represented as $\Sigma_5 = mcx^5 + \Sigma(x^1, x^2, x^3, x^4)$. Consequently, the wave vector in the fifth dimension corresponds to the mass and the normalized vector - the number of unit mass. The calculated value $\sqrt{S} = 293$ for the curve in Figure 2, probably corresponds to the element with the highest atomic number ^{293}Ei according to [19].

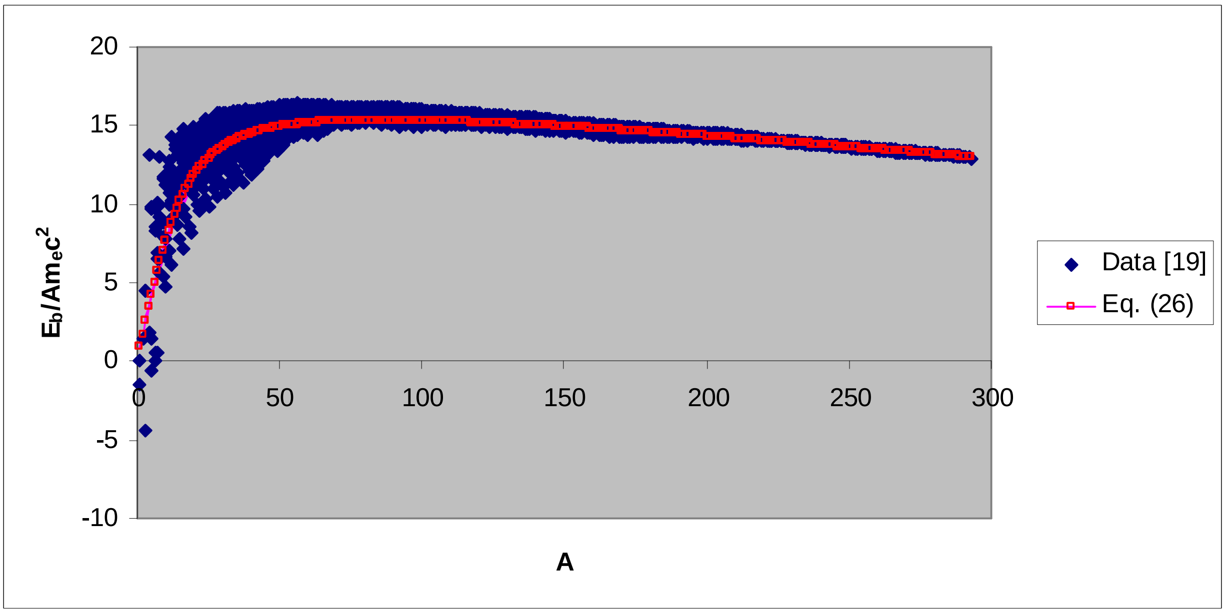

Figure 2: The binding energy per nucleon as a function of mass number according to [19] and calculated according to equation (26).

The average value of magnetic charge $u = 0.9986$ indicates a high degree of correlation of the nucleons in the nucleus. The resulting value of the interaction parameter $b_0 = (2a\, m_e / m_p (1 - 2a))^2 \approx 3.5 \cdot 10^{-8}$ gives the average value of the angular moment $\pm l = a \approx 0.478744$.

The ground state energy of the neutron shell is determined by equation (25), in which we set $X = N$. The parameters of the neutron shell do not coincide with the parameters of the nucleon shell, but varies for each element. Using together expressions the binding energy of the nucleon and the neutron shell we can quite accurately describe the energy of isotopes – Figure 3. The computational model is constructed as follows. Suppose that, based on equation (26) for the nucleon shell failed to accurately determine the binding energy of one isotope of an element. Without loss of generality we can assume that this isotope contains the minimum number of neutrons. Then the binding energy of all other isotopes of an element defined as follows

$$\frac{E(N,Z)}{A} = \frac{E_{0a}^{A}(N_{\min}, Z)}{Z + N_{\min}} + \frac{E_{0a}^{N}(N,Z)}{Z+N} - \frac{E_{0a}^{N}(N_{\min}, Z)}{Z + N_{\min}} \tag{27}$$

Model (25) - (27) contains the arbitrary choice of the interaction parameter, which does not depend on the nuclear charge. As an example, Table 3 shows the model parameters, calculated according to [19] for isotopes of gold.

Table 3: Parameters eq. (25), (27) calculated for isotopes of gold.

Sb_0	u	$S^{3/2}b_0u$	$1-u^2$	$Sb_0(1-u^2)$	$S^{1/2}$
0,005	0,9936	3,60	0,012759	6,38E-05	724,6377
0,05	0,9994	11,49	0,0012	6E-05	229,938
0,56	0,999947	38,05	0,000106	5,94E-05	67,94289
1	0,99997	50,98	5,94E-05	5,94E-05	50,98151
5	0,999994	114,10	1,19E-05	5,94E-05	22,82014
10	0,999997	161,42	5,94E-06	5,94E-05	16,14205

In contrast, the parameter $Sb_0(1 - u^2) = p(Z)$ is almost constant for a given element, but it depends on the magnitude of the charge. This dependence is monotonous - the interaction parameter decreases with increasing charge. This result indicates that the proton and neutron shell nuclei interact with each other.

The ground state energy of the proton shell is determined by equation (25), in which we set $X = Z$. The computational model in this case is constructed similarly to (27) with the replacement $N \to Z$. As a result, we find:

$$\frac{E(N,Z)}{A} = \frac{E_{0a}^{A}(Z_{\min},N)}{N+Z_{\min}} + \frac{E_{0a}^{Z}(Z,N)}{Z+N} - \frac{E_{0a}^{Z}(Z_{\min},N)}{N+Z_{\min}} \tag{28}$$

Using together equation (25) and (28) we can determine the binding energy of nuclides with a given number of neutrons - Figure 4. In this case also there is arbitrariness in the choice of the interaction parameter, which is practically independent of the number of neutrons of the nucleus. As an example, Table 4 shows the model parameters, calculated according to [19] for N = 111.

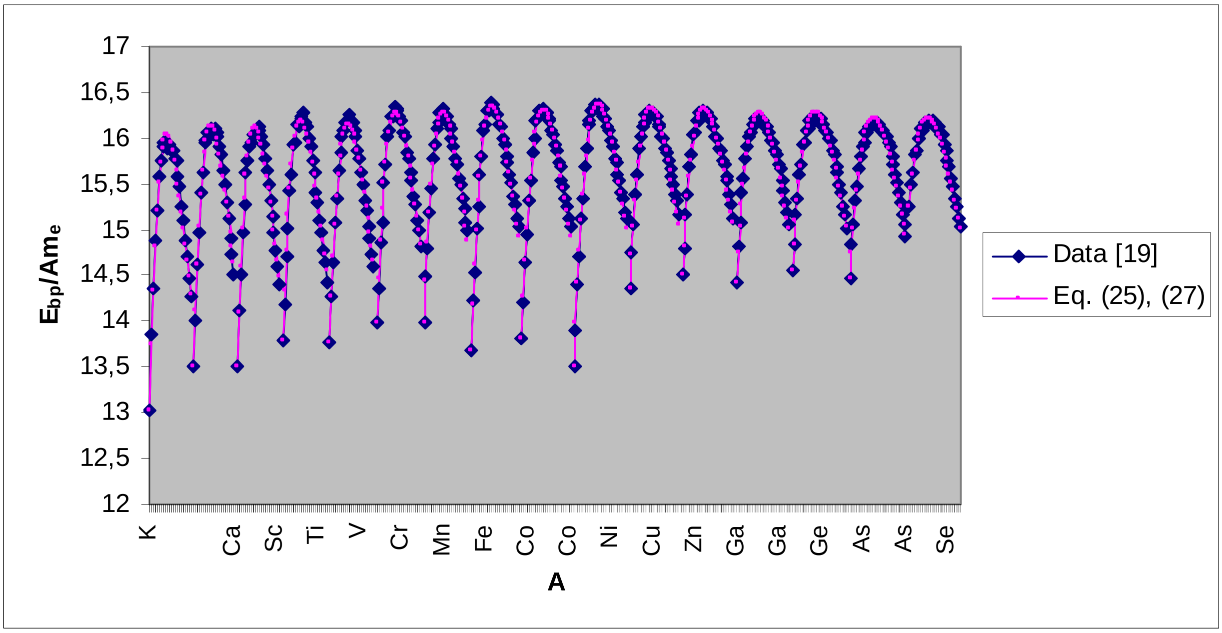

Figure 3: Binding energy per nucleon as a function of mass number calculated for a number of isotopes of chemical elements on the model (25), (27) and according to [19].

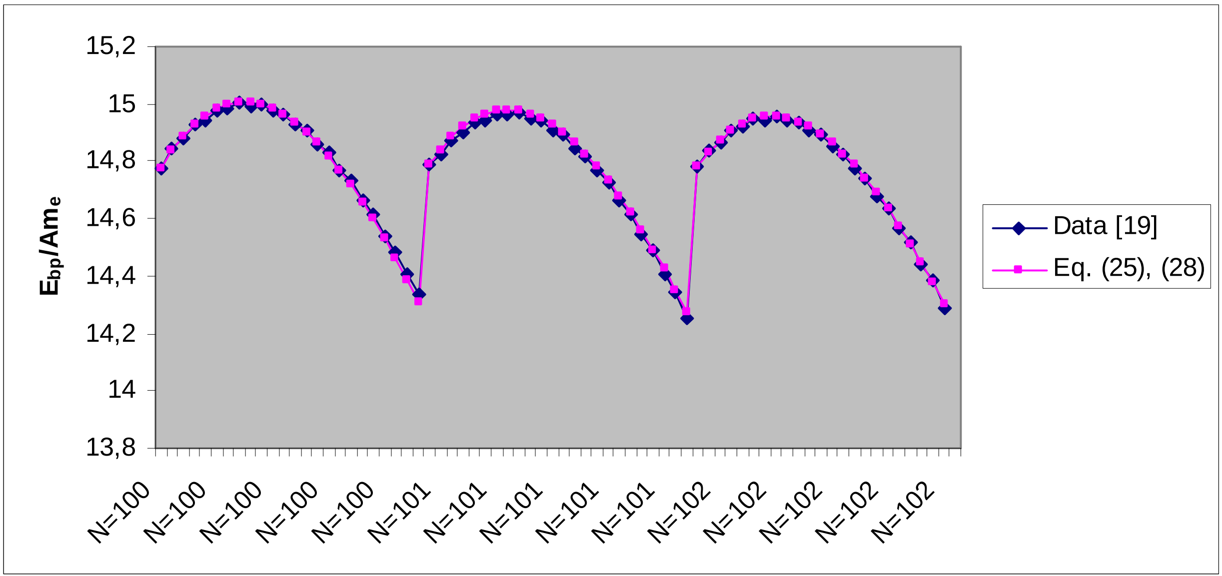

Figure 4: Binding energy per nucleon as a function of mass number calculated for N = 100, 101, 102 on the model (25), (27) and according to [19].

Table 4: Parameters eq. (25), (28) calculated for N=111.

Sb_0	u	$S^{3/2}b_0u$	$1-u^2$	$Sb_0(1-u^2)$	$S^{1/2}$
0,01	0,9842	19,13285	0,03135	0,000314	1944
0,05	0,9971	45,54254	0,005792	0,00029	913,5
0,1	0,99855	62,30952	0,002898	0,00029	624
0,5	0,99971	135,5607	0,00058	0,00029	271,2
1	0,999856	196,3717	0,000288	0,000288	196,4
2	0,999928	277,16	0,000144	0,000288	138,59
3	0,999952	339,2567	9,6E-05	0,000288	113,091
6	0,999976	479,3285	4,8E-05	0,000288	79,89
10	0,999986	602,6913	2,9E-05	0,00029	60,27

The question arises, what mode of interaction is realized in atomic nuclei - with a large or small value of the parameter of interaction Sb_0? To answer this question, we calculate the characteristic scale r_0 appearing in the equations (20). Solving the third equation (20) with respect to size, we find from (24) - (25) that

$$r_0 = \frac{\hbar}{m_e c\sqrt{P^2 - S - E^2}} = \frac{\hbar\sqrt{Sb_{nl}^X}}{m_e cE} = \frac{\hbar}{m_e cP}\frac{1+Z^2}{uZ} = \frac{1+Z^2}{k_z uZ} \qquad (29)$$

In Fig. 5 shows the standard nuclei size and model scale r_0 for the isotopes of gold. These data imply that the model scale for gold will be consistent with the real size of nuclei at $Sb_0 \approx 1$. The standard size of the nuclei depends on the number of nucleons [8, 20]:

$$r(A) = r_A A^{1/3}, \ r_A = (1.12 \div 1.4)10^{-15} m .$$

Consequently, the proton shell is implemented with the value of the interaction parameter $Sb_0 \approx 1$. It was found that the orbital angular momentum has a value close to ½ for $Sb_0 \approx 1$ - Table 5. Thus, the initial scalar field acquires the charge and mass, and spin, like fermions, which clarifies the nature of the electron spin.

Note that in the case of $Sb_0 = 1$ a parameter characterizing the interaction of the magnetic type, $Sb_0(1-u^2) = f(N)$ decreases monotonically with increasing numbers of neutrons by a power law, and a parameter $S^{1/2} = \hbar k_r / m_e c$ characterizing the motion of matter in the fifth dimension, almost linearly dependent on the number of neutrons - Fig. 6.

Table 5: Parameters of equation (26), calculated for some isotones.

N	*Sb₀*	*u*	*S^1/2*	*f(N)*	*a*	*L*
2	1	0,8	105	0,36	0,499583	0,499583
3	1	0,925	67	0,144375	0,499734	0,499734
4	1	0,964	55,9	0,070704	0,499778	0,499778
5	1	0,979	50,8	0,041559	0,499798	0,499798
10	1	0,9942	69,4	0,011566	0,499724	0,499724
20	1	0,9981	84,9	0,003796	0,499663	0,499663
30	1	0,9987	97	0,002598	0,499614	0,499615
40	1	0,99937	111	0,00126	0,499559	0,499559
50	1	0,99955	105,5	0,0009	0,499581	0,499581
60	1	0,999656	119,5	0,000688	0,499525	0,499526
70	1	0,99972	131	0,00056	0,499479	0,49948
80	1	0,99976	154	0,00048	0,499388	0,499389
90	1	0,9998	141	0,0004	0,49944	0,499441
110	1	0,99985	163	0,0003	0,499352	0,499354
120	1	0,999873	243	0,000254	0,499034	0,499038
130	1	0,999877	185	0,000246	0,499265	0,499267
150	1	0,999906	239	0,000188	0,49905	0,499054

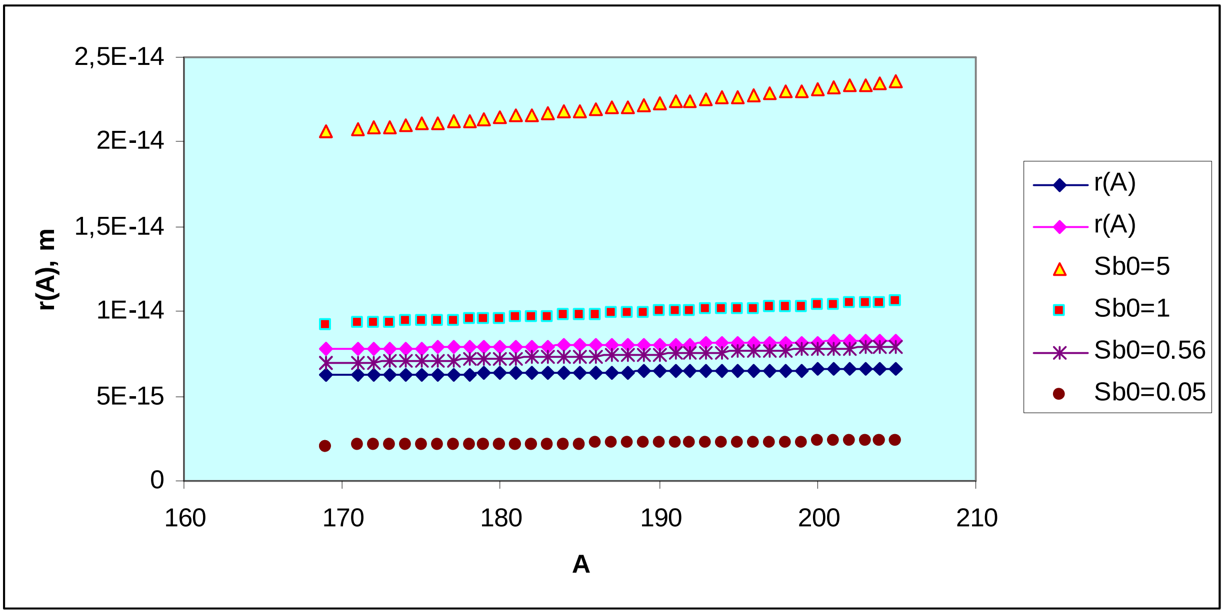

Figure 5: Gold nuclei isotope radius and the characteristic scale of the model as function of mass number.

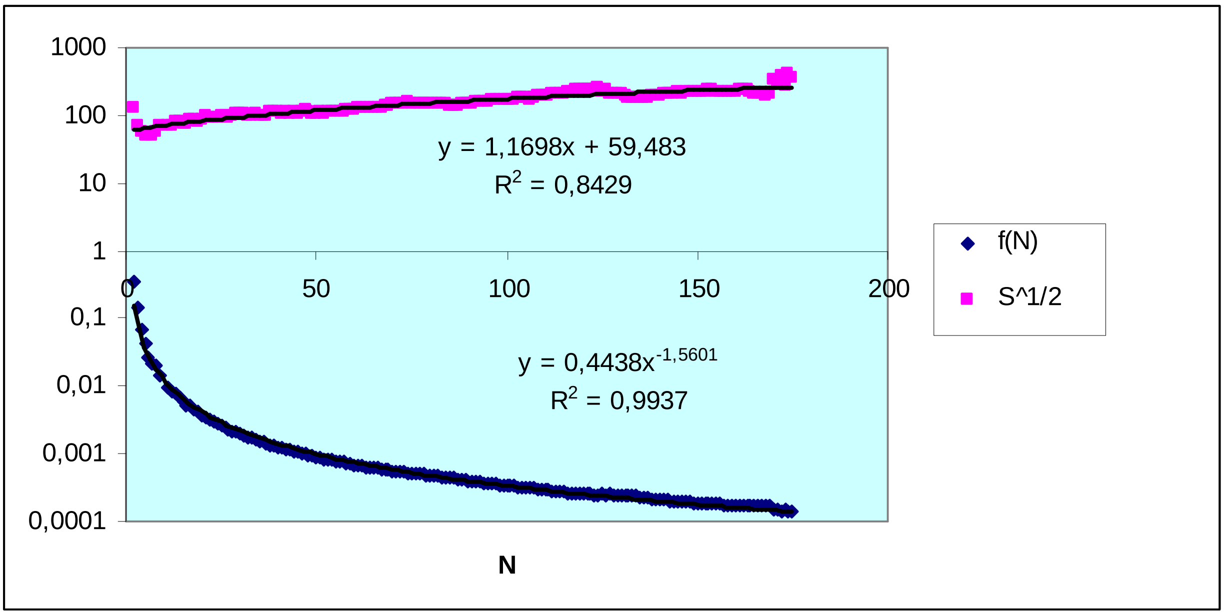

Figure 6: Interaction parameter $f(N) = Sb_0(1-u^2)$ and parameter $S^{1/2} = \hbar k_r / m_e c$ characterizing the motion of matter in the fifth dimension as a function of number of neutrons at $Sb_0 = 1$.

We have shown that the nuclear shells, consisting of nucleons, neutrons and protons, respectively, allow modelling the binding energy as a function of neutron, proton and mass number. Data shown in Figure 6, and Table 5, indicate that the proton shell interacts with the neutron shell.

The question arises which of the nuclear shell more significant effects on the properties of atomic nuclei and chemical elements? The neutron and proton shell can equally be used to model the binding energy, and it is consistent with the principle of isotopic symmetry of nuclear forces. However, the electron shells depend on the number of proton nuclei, which implies that the proton shell has a greater influence on the atomic scale, rather than a neutron shell. This result is compatible with modern formulation of the periodic law in the form (2).

Electron shell

We consider the general expression of energy (23) in the case of the proton shell, and subject to full screening of magnetic charge, i.e. put in the right-hand side of (23) $X = Z, u = 0$. The result is

$$E_{nl}^{Z} = \frac{\pm\sqrt{(Sb_{nl}^{Z}+1)(P^2-S)}}{(Sb_{nl}^{Z}+1)}, \quad b_{nl}^{Z} = \frac{4Z^2(a\,m_e/m_p)^2}{\left(n+1-2\sqrt{l^2-SZ^2(a\,m_e/m_p)^2}\right)^2} \tag{30}$$

On the other hand, in the case of hydrogen atom there is the Sommerfeld-Dirac formula for the energy of a relativistic electron [17-18, 21]

$$E_e = \frac{m_e c^2}{\sqrt{1+\dfrac{a^2 Z^2}{(n_r+\sqrt{n_f{}^2-a^2Z^2})^2}}} \tag{31}$$

Comparing (30) and (31), we find that for agreement of these formulas should be put

$$E_{nl}^{Z}>0,\quad P^{2}=1+S,\quad S=(m_{p}/m_{e})^{2},\quad n_{r}=(n+1)/2,\quad n_{f}=l \tag{32}$$

Note that a difference in a sign of the radical $\sqrt{l^{2}-a^{2}Z^{2}}$ arises due to the choice of the sign of the parameter in the expression of the wave function (18), where in general case, we should put $a=\pm\sqrt{l^{2}-k_{u}}$. In the nuclei structure problem we have chosen a positive sign, whereas for atomic shells taken negative sign. In the latter case we find from (32) that the first equation (30) coincides with the Sommerfeld-Dirac equation (31):

$$E_{nl}^{Z}=\frac{E_{e}}{m_{e}c^{2}}=\frac{1}{\sqrt{1+\dfrac{a^{2}Z^{2}}{(n_{r}+\sqrt{n_{f}^{2}-a^{2}Z^{2}})^{2}}}} \tag{33}$$

Thus, we have shown that the expression (23) is universal one. In the region $u\approx 1$ this expression describes the energy of the nucleons in the nuclei, while at the condition $u=0$ it describes the energy of relativistic electrons in atomic shells. There is a special case of states of the hydrogen atom - hydrino [10, 17-18], which is also described by (23).

The first terms in the expansion of (33) in powers of small parameter $(aZ)^{2}<<1$ describing the energy levels of hydrogen atoms, including hydrogen, in this case we have [21]

$$E_{nl}^{Z}-1=\frac{E_{e}}{m_{e}c^{2}}-1=-\frac{(aZ)^{2}}{2(|n_{f}|+n_{r})^{2}}\left(1+\frac{a^{2}Z^{2}}{|n_{f}|+n_{r}}\left(\frac{1}{|n_{f}|}-\frac{3}{4(|n_{f}|+n_{r})}\right)\right)+\ldots \tag{34}$$

Equation (34) describes the X-terms, the quadratic dependence on the nuclear charge has been determined experimentally by Moseley [4], which served as the basis for the creation of quantum mechanics and modern form of the periodic law.

Note that the general expression (30) contains free parameters, which in modern quantum theory take the particular values of (32). Given these values, we find that the expression (33) describes not only the energy of the bound states of an electron in a hydrogen atom (34), but the energy of a free electron due to its rest mass. Equation (11) describes the motion of a scalar field, which does not possess, nor charge nor mass nor spin. Consequently, we have shown that the electron rest mass, its charge and spin motion due to a scalar massless field in the five-dimensional space with a special metric (10), depending on the charge and mass of the central core.

Nuclear shell and periodic trends

Currently, there are at least 473 versions of the periodic table [22]. A modern kind of periodic table with nuclides in the plane (N, Z) included basic properties of nuclei [26]:

1) Spin and parity Jp ;

2) Mass excess $\Delta = M_A - A$ calculated with respect to isotope ^{12}C;

3) Half-life, or width, and abundance of an element in nature (%);

4) Decay mode.

Note, the periodic table can be linked to information theory [23-24] based on an idea of chaos and order, entropy and information connected with complexity of the object, which is defined as the minimum length of algorithmic programs needed to get some Y of X [25]. In this regard, we note that in this theory, an algorithm for obtaining the atomic energy levels is no different from the algorithm of obtaining the energy levels of the nucleus. Consequently, it is assumed that an atom is arranged not more complicated than nucleus and a nucleus is no more complex than an atom.

Using the theory developed above, we can determine the order of elements and isotopes, all of them, considering both the process of filling of proton and electron shells. The increase in nuclear charge by one proton leads to a change in nuclear mass and binding energy of nucleons and electrons, which in turn leads to a change in the chemical and physical properties of chemical elements. As shown in figure 6 parameters of proton shell depend on the number of neutrons. Using the equation of the trend for the data in Fig. 6, we can determine how to change the parameters $f(N)$, $S(N)$ with respect to the trend line – see Figures 7-8:

$$
\begin{aligned}
Vf(N) &= f(N) * N^{1.5601} / 0.4438 \\
VS &= S^{1/2}(N)/(1.1698 N + 59.483)
\end{aligned}
\tag{38}
$$

These changes have a clear periodic component that allows to select in the table of nuclides their own periods. These periods can be associated with neutron shell, which, according to the nuclear shell model [9], filled in the same way as the filled electron shells. In such a case should be allocated the magic numbers 2, 8, 20, 28, 50, 82, 126, 184. Indeed, according to Fig. 7, we can identify periods associated with the filling of shells with neutron number between 2 and 8, 50 and 82, 82 and 126, 126 and 170, which corresponds to the 2nd, 6th, 7th and eighth shell. Magic numbers can be compared with the same points in the curves in Figure 7. The relevant data are collected in Table 7. These numbers do not match exactly the magic numbers, although they are close to them.

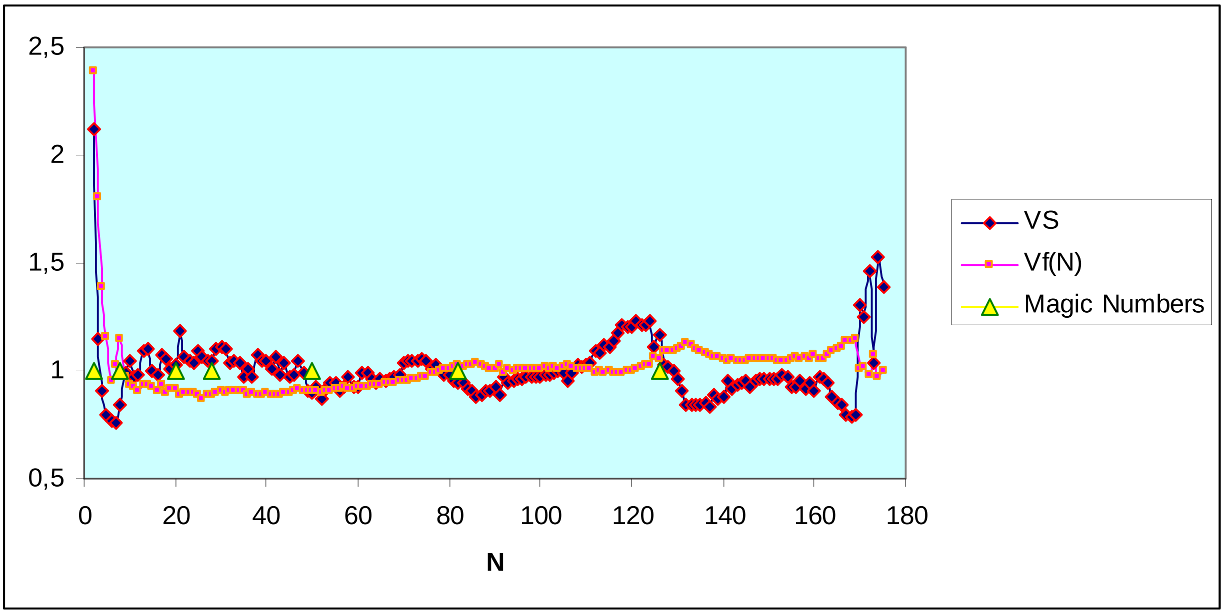

Figure 7: Fluctuations in the model parameters (28) over trend lines: $Vf(N) = f(N) * N^{1.5601} / 0.4438$; $VS = S^{1/2}/(1.1698 N + 59.483)$

Table 7: Number of neutrons corresponding to maximal and minimal values of parameters (38) and magic numbers.

VSmax	2	21	74	124	174
VSmin	7	52	86	133	169
Vfmax	2,8		87	133	169
Vfmin	6	26			174
Magic numbers	2, 8	20, 28, 50	82	126	184

Finally we note, that standard theory of nuclear shell [9] in which the actual motion of the nucleons in the nucleus is modeled based on the model of a quantum harmonic oscillator, is consistent with this theory, in which the motion of the nucleons is modeled on the basis of a unified model of nuclear and atomic field. However, for light nuclei, there is a discrepancy that can be seen from the data shown in Fig. 8. We can assume that in the case of third, fourth, and fifth shell periodic law is more complicated than that predicted by the standard theory of nuclear shells.

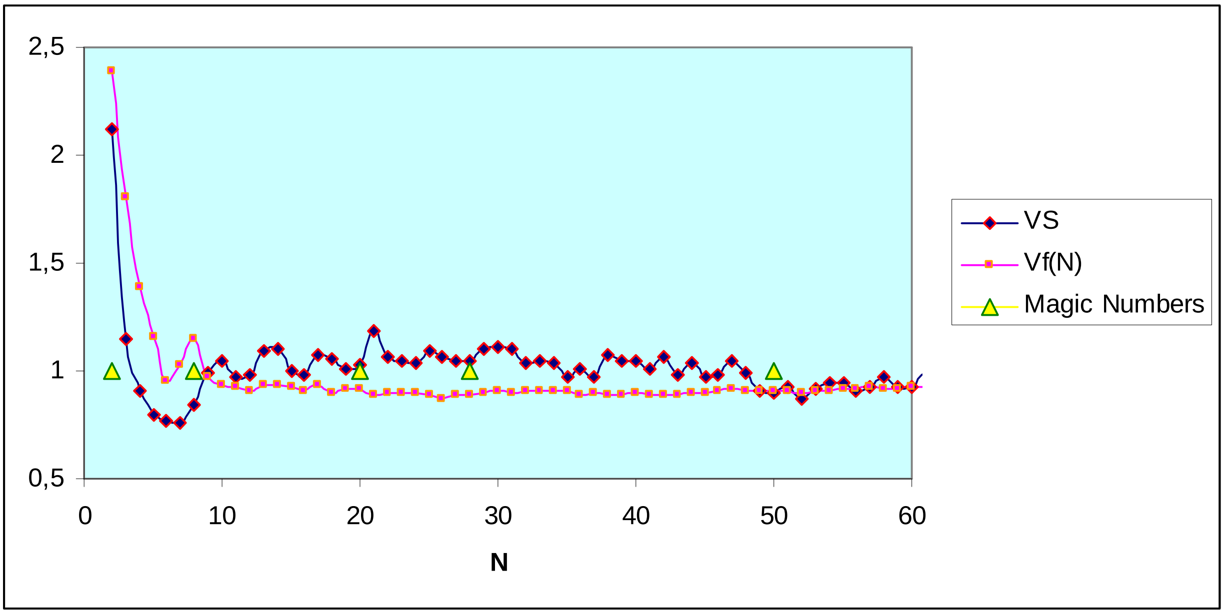

Figure 8: Fluctuations in the model parameters (28) over trend lines in the case of light nuclei.

Finally we can suggest that periodic properties of chemical elements depend on the number of protons (charge), and on the number of neutrons (mass) as well in according with original Mendeleev's periodic law.

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